THE EFFECT OF BOUNDARY CONDITIONS ON THE GENERAL MECHANICAL BEHAVIOR OF SOFT MATERIALS

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ABSTRACT

In this work, a model is proposed extending the basic concept of particle of continuum mechanics combining electric properties. This, as Gaussian statistics denoted background for the single entropic chain segments, is used for the derivation of the Gaussian network models consisting of a highly cross-linked network of such Gaussian chains providing a link of the micromechanical polymer model to macroscopic scales. In particular, the interface of soft material is separated from the whole body as a single phase. The interaction between different phase is studied when the proper parameters are selected, electric breakdown, flexoelectricity effect even the plasmon-exciton coupling.

1 INTRODUCTION

Compared with traditional materials the advantages of soft materials are obviously, light weight, rapid response and silent operation [1]. However, a high voltage is needed to run the actuators made by soft materials when comparing with the actuators made by shape memory materials and piezoelectric materials, also low energy density [2]. are generally the applications of soft materials are limited due to their failure mechanical both for pure mechanical (loss of tension, rupture by stretch and aging) and electromechanical loading (electric breakdown, electromechanical instability)[3]. These limitations can be improved using composite materials, pre-stretch or modified structure [4].

Many attempts has been tried to modify the drawback mentioned above using the new structures: symmetric or anti-symmetric structure [5][6] rolled or stacked structure and sphere like structure [7]. The investigation shows that the bent behavior and boundary condition is still the key to control the equilibrium state, the equilibrium state can be found ranging from 40° to 40°. Rolled or stacked structures have extends the application of the actuators from plane to space due to their multiple degree-of-freedom. With suitable values of the parameters, the sphere structure obtain giant voltage-induced expansion of area by 1692%, far beyond the largest value reported in the literature [7].

The macro mechanical properties have been found originates from the micro structure of polymers, including spatial distribution, interfacial reaction and chemical modification [8]. Based on the analysis of micro structure [9], several multi scale models have been proposed, such as 3 chain model, 8 chain model and tetrahedral model [10]. Beyond that, the hyperelastic models aimed to capture the limiting stretch of polymer chain are always used to mechanical behavior of soft polymer, such as Ogden model and Gent model.

2 THE RESTRICTION BEHAVIOR OF BOUNDARY ON GAUSSIAN CHAIN

In this section, the effective polymer chain length L is introduced to characterize the effect of polymer chain on the mechanical behavior. We start by considering the polymer chain from a point P inside the system and far from any boundaries (both inside and outside), and this condition can be regard as random distribution. Thus the angle $\theta_i(L)$ of polymer chain along i = x, y and z direction can be expressed as:

$$0 \le \theta_i(L) < 2\pi, \quad i = \text{x, y and z} \tag{1}$$

$$\theta_x(L):\theta_y(L):\theta_z(L)=1:1:1 \tag{2}$$

where θ is the space rotation angle. The space rotation angle $\theta_i(L)$ is more complicated when the point P is near the boundary. For a better description, a local coordinate system ($\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$) with $\mathbf{e}_1 = -\mathbf{n}$ is used ($\mathbf{n}$ is the normal vector of the boundary). On the basis that the boundary is smooth, the simple relationship can be read:

$$\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij} \quad i, j = 1,2,3 \tag{3}$$

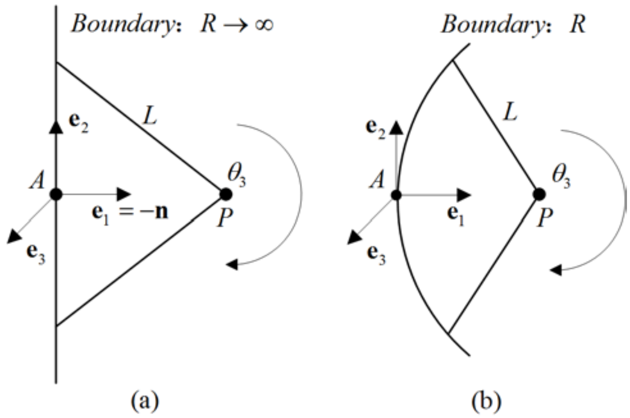


Figure 1. Restriction of boundary condition on the distribution of polymer chain.

Taking the local boundary as a plane, as shown in figure 1 (a), the radius of curvature $R \to \infty$, the space rotation angle on point P (X_1, 0, 0) can be expressed as ($X_1 < L$):

$$\begin{aligned}
&\mathbf{e}_1 : 0 \le \theta_1(X_1;L) < 2\pi \\
&\mathbf{e}_2 : \arccos\left(\frac{X_1}{L}\right) \le \theta_2(X_1;L) < 2\pi - \arccos\left(\frac{X_1}{L}\right) \\
&\mathbf{e}_3 : \arccos\left(\frac{X_1}{L}\right) \le \theta_3(X_1;L) < 2\pi - \arccos\left(\frac{X_1}{L}\right)
\end{aligned} \tag{4}$$

where the restricted angle θ' can be denoted by $\cos(\theta') = X_1/L$. One can easily concluded that the space rotation angle of a boundary point ($X_1 = 0$) $0 \le \theta_1 < 2\pi;\ 0 \le \theta_2 < \pi;\ 0 \le \theta_3 < \pi$ from equation (4). This is means that the angle is limited by the boundary condition.

When we consider the general situation: the local boundary is curved surface and can be noted by two radius of curvatures R_2 and R_3 along $\mathbf{e}_2$ and $\mathbf{e}_3$ direction, respectively. Definition of curvature reads:

$$R_2 = \left|\frac{ds_2}{d\varphi_2}\right|; R_3 = \left|\frac{ds_3}{d\varphi_3}\right| \tag{5}$$

where s and φ are the local arc length and corresponding angle along $\mathbf{e}_2$ and $\mathbf{e}_3$ direction, as shown in figure 1 (b). Similarly, the are characterized using point (X_1, 0, 0):

$$\begin{aligned}
&\mathbf{e}_1 : 0 \le \theta_1(X_1;L) < 2\pi \\
&\mathbf{e}_2 : \arccos\left(\frac{R_2}{L}\right) + \varphi_2 \le \theta_2(X_1;L) < 2\pi - \arccos\left(\frac{R_2}{L}\right) - \varphi_2 \\
&\mathbf{e}_3 : \arccos\left(\frac{R_3}{L}\right) + \varphi_3 \le \theta_3(X_1;L) < 2\pi - \arccos\left(\frac{R_3}{L}\right) - \varphi_3
\end{aligned} \tag{6}$$

We should notice that in any situation, the restricted behavior described using angle are equals to that using the global coordinate $\mathbf{X}$, and denoted by $\Omega_{res}(\mathbf{X}, L)$ which is a function of the reference state of the system. The probability of random walking in one dimension without restriction can be expressed as Gaussian distribution:

$$p\left(x,N\right)=\frac{1}{\sqrt{2\pi NL^2}}e^{-\frac{x^2}{2NL^2}} \quad (7)$$

where $x = NL$ is the walking distance, N is the walking step. The extended Gaussian distribution obtained considering an restricted distance l which means the probability equals to zeros when $x < l$, and expressed as follows:

$$\begin{cases} p\left(x<l,N\right)=0 \\ p\left(x\geq l,N\right)=\frac{1}{\sqrt{2\pi NL^2}}e^{-\frac{x^2}{2NL^2}}+\frac{1}{\sqrt{2\pi NL^2}}e^{-\frac{(x-2l)^2}{2NL^2}} \end{cases} \quad (8)$$

Equation (15) is not the exact value due to the assumptions in [Treloar L R G], and the emphasize is that the restricted behavior are considered. When bring equations (4 - 6) into equation (8) and extend the equations to three dimension situation.

$$\begin{cases} p\left(X<\Omega_{res},N\right)=0 \\ p\left(X\geq \Omega_{res},N\right)=\prod_{i=1}^{3}p\left(X_i\geq \Omega_{res,i},N\right) \end{cases} \quad (9)$$

3 THE IMPROVED MECHANICAL MODEL OF SOFT MATERIALS

The mechanical constitutive behavior of materials according to [11], which means the relation between stress σ and strain $\varepsilon = \lambda - 1$ can be summarized as follows:

$$\sum_{k=0}^{m} p_k \frac{\mathrm{d}^k\sigma}{\mathrm{d}t^k}=\sum_{k=0}^{n} q_k \frac{\mathrm{d}^k\varepsilon}{\mathrm{d}t^k} \quad (10)$$

where p and q are the parameters of materials. Equation (10) is the general expression for the modified kelvin-voigt model. Applying the hydrostatic pressure p into equation (10), we can rewrite the stress-strain relationship as:

$$\sum_{k=0}^{m} p_k \frac{\mathrm{d}^k\left(\sigma_{ij}+P_{hy}\delta_{ij}\right)}{\mathrm{d}t^k}=\sum_{k=0}^{n} q_k \frac{\mathrm{d}^k\varepsilon_{ij}}{\mathrm{d}t^k},\ i=x,y \text{ and } z \quad (11)$$

where δ_{ij} is the kronecker delta ($\delta_{ij}=1$ if $i = j$, and $\delta_{ij}=0$ otherwise). In equation (11), only normal stress is considered (shear stress can be ignored) as the main mode that external works, shear stress is not appropriate for the application of the material.

Many studies have attempted to separate the time dependent behavior from mechanical models by introducing a time function, as can be seen in **[**11**]**. Similarly, the equivalent modulus or other expressions derived from energy conservation method.

$$\boldsymbol{\sigma}+P_{hy}\mathbf{I}=\mathbf{E}_{equ}\boldsymbol{\varepsilon} \quad (12)$$

where E_{equ} denotes the equivalent elastic modulus and can be derived from equation (12), $\mathbf{I}$ is the unit tensor. When we consider *np* models connected in parallel, the probability in section 2.2, the relationship between stress and strain of the point $\mathbf{X}$ can be expressed as:

$$\begin{aligned} &\boldsymbol{\sigma}+P_{hy}\mathbf{I}=n\mathbf{P}\mathbf{E}_{equ}\boldsymbol{\varepsilon} \\ \text{or } &\boldsymbol{\sigma}+P_{hy}\mathbf{I}=n_{in}\mathbf{P}\mathbf{E}_{equ}\boldsymbol{\varepsilon} \end{aligned} \quad (13)$$

where **p** is the matrix expression of equation (11).

4 SIMULATION OF CIRCULAR DIELECTRIC ELASTOMER

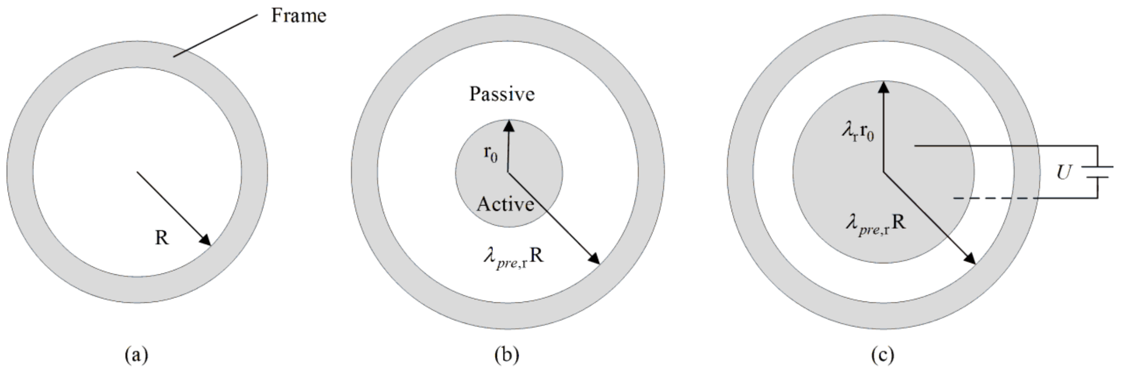
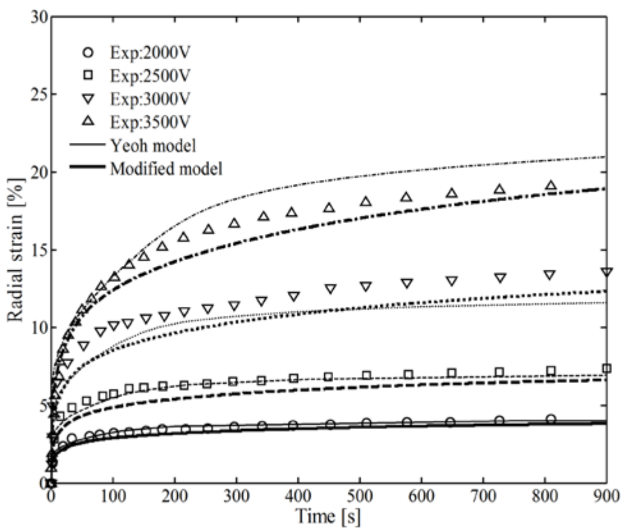


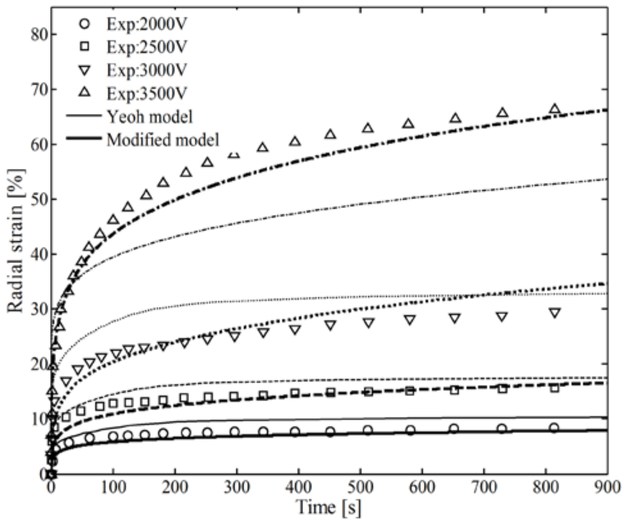
Figure 2.Schematic of a circular dielectric elastomer actuator in different state: (a) Reference state. (b) Pre-stretched state. (c) Activated state.

In reference state, a circular elastomer membrane with radius R is fixed on a circular frame (figure 2(a)). Then the membrane is pre-stretched biaxially with stretch ratio $\lambda_{pre,r}$ to a level $\lambda_{pre,r}R = 75\text{mm}$ in pre-stretched state. Then the active center area is coated with compliant electrodes on two sides, $r_0 = 7.5\text{mm}$ (figure 2(b)). In activated state, an external voltage *U* is applied on the active area, then the radius of active area changed into $\lambda_r r_0$, then the radial strain can be wrote as λ_r -1 (figure 2(c)).

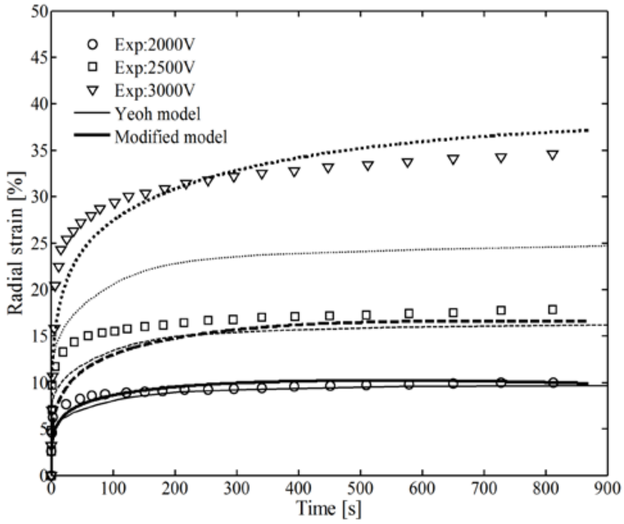
With different pre-stretched ratio $\lambda_{pre,r}$ and applied voltage *U*, the relationship between time and radial strain of active area λ_r -1 are plotted in figure 3.



(a)



(b)



(c)

Figure 3.Fit the improved model, Yeoh model [12] to the circular dielectric elastomers: (a) $\lambda_{pre,r}$ = 3. (b) $\lambda_{pre,r}$ = 4. (c) $\lambda_{pre,r}$ = 5.

From figure 3(a), figure 3(b) and figure 3(c), some conclusions can be drawn as follows. Subjected to a same voltage, the radial strain is increased with pre-stretched level. Similarly, when we keep the same pre-stretched level, the radial strain are increased with the increasing the voltage applied. The radial strain changed temporality and quickly when the voltage applied ($t < 300$), then remained to a same level (t □ 300) almost constant for a long time.

The relative error of Yeoh model are 18% and 21% for test $\lambda_{pre,r} = 4$ $U = 3500\text{V}$ and test $\lambda_{pre,r} = 5, U = 3000\text{V}$, respectively (figure 3(b) and figure 3(c)). However, the relative errors of the modified model are 4.6% and 6.2% which are much more acceptable. Obviously, the simulation results of the improved model are more reasonable compared with Yeoh model.

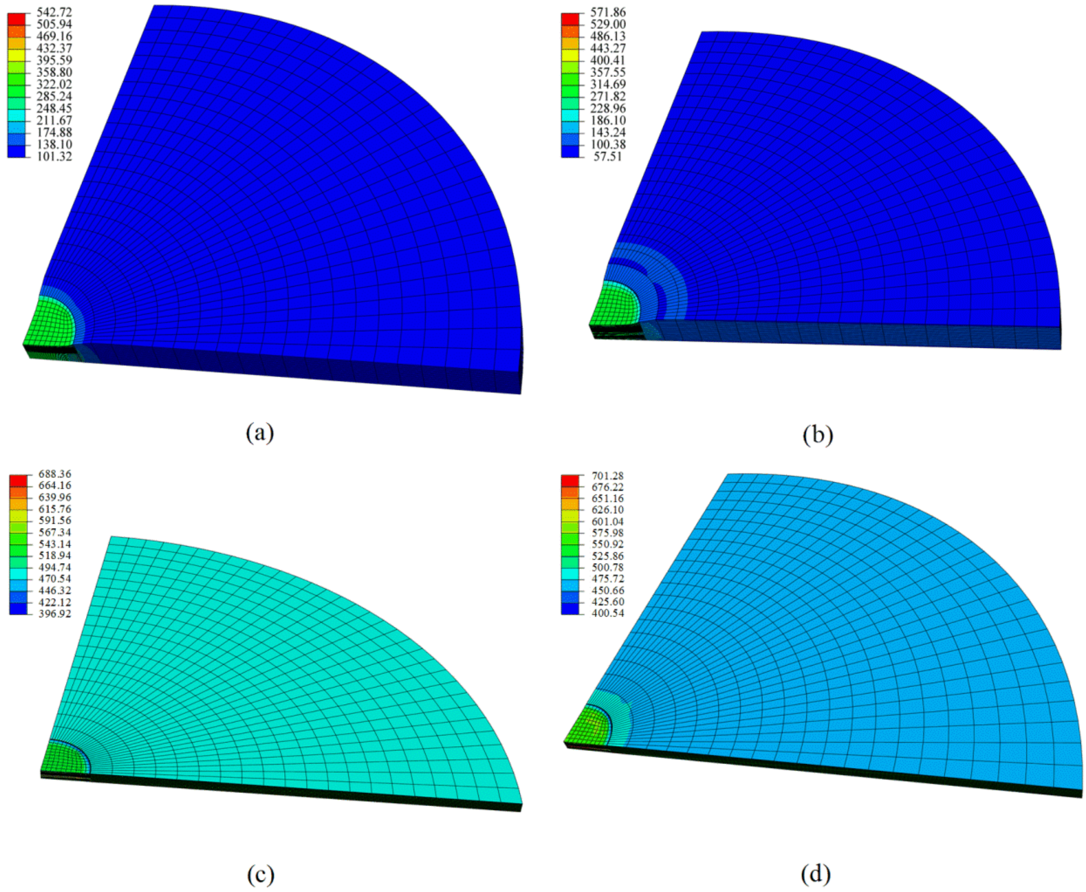


Figure 4. simulation results of circular dielectric elastomer at time t = 300s (a) Yeoh model $\lambda_{pre,r} = 3$ and $U = 2000\text{V}$;(b) Modified model $\lambda_{pre,r} = 3$ and $U = 2000\text{V}$;(c) Yeoh model $\lambda_{pre,r} = 5$ and $U = 3000\text{V}$; (d) Modified model $\lambda_{pre,r} = 5$ and $U = 3000\text{V}$;.

In figure 4, we show four states during the simulation including the Yeoh model and the modified model. As for the lower pre-stretched level and voltage ($\lambda_{pre,r} = 3$ and $U = 2000\text{V}$) the simulation results (both the stress and the strain) of the two models are close as can be seen from figure 4(a) and (b). However, at higher pre-stretched level and voltage ($\lambda_{pre,r} = 5$ and $U = 3000\text{V}$) the boundary of the activated area extending with time more quickly. Differently, the boundary movement using the modified model is more quickly than Yeoh model, as can be seen from figure 4(c) and (d).

5 CONCLUSIONS

In summary, we have developed a model based on the continuum mechanics. The general electromechanical behavior of soft materials is characterized by extending the Gaussian distribution of polymer chain. Generally, inside the system the chains are regard as random distribution, however, near any surfaces (inside or outside) the distribution of the chains are restricted.

The simulation results shows that when the deformation increase, the surface part of the soft materials has the potential to increase the instability and further deformation due to the low corresponding elastic modulus. More important, the dielectric couple effect is increased by this phenomenon comparing with isotropic assumption. As for a complex system, is seems that the electric field is a function of the surface characteristics which is changing during time and coupling with the deformation of the system.

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REFERENCES

[1] S. Qu, Z. Suo, A finite element method for dielectric elastomer transducers, *Acta Mechanica Solida Sinica*, **25**(5),2012 ,pp. 459-466.

[2] N. Cohen, K. Dayal, Electroelasticity of polymer networks, *Journal of the Mechanics and Physics of Solids*, **92**, 2016, PP. 105-126.

[3] S. J. A. Koh, X. Zhao, Z. Suo, Maximal energy that can be converted by a dielectric elastomer generator, *Applied Physics Letters*, **94**(26), 2009, 262902.

[4] G.Kofod, The static actuation of dielectric elastomer actuators: how does pre-stretch improve actuation? *Journal of Physics D: Applied Physics*, **41**(21), 2008, 215405.

[5] J. Zhao, J. Niu, D. McCoul, et al, Phenomena of nonlinear oscillation and special resonance of a dielectric elastomer minimum energy structure rotary joint, *Applied Physics Letters*, **106**(13), 2015, 133504.

[6] G. Buchberger, B. Hauser, J. Schoeftner, et al, Temporal change in the electromechanical properties of dielectric elastomer minimum energy structures, *Journal of Applied Physics*, **115**(21), 2014, 214105.

[7] T. Li, C. Keplinger, R. Baumgartner R, et al, Giant voltage-induced deformation in dielectric elastomers near the verge of snap-through instability, *Journal of the Mechanics and Physics of Solids*, **61**(2),2013 , pp. 611-628.

[8] B. Pukánszky. Interfaces and interphases in multicomponent materials: past, present, future,.*European Polymer Journal*, **41**(4), 2005,pp. 645-662.

[9] W. Kuhn, F. Grün, Beziehungen zwischen elastischen Konstanten und Dehnungsdoppelbrechung hochelastischer Stoffe, Kolloid-Zeitschrift, *Colloid & Polymer Science*, **101**(3), 1942, pp. 248-271.

[10] L. R. G. Treloar, The elasticity of a network of long-chain molecules.—III. *Transactions of the Faraday Society*, **42**, 1946, pp. 83-94.

[11] P. Lochmatter, G. Kovacs, M. Wissler. Characterization of dielectric elastomer actuators based on a visco-hyperelastic film model. *Smart Materials and Structures*, ***16***(2), 2007, 477.

[12] M. Wissler, E. Mazza, Mechanical behavior of an acrylic elastomer used in dielectric elastomer actuators. *Sensors and Actuators A: Physical*, ***134***(2), 2007, 494-504.