ON THE STOKES-BRINKMAN COUPLING WITH ANISOTROPIC NAVIER SLIP AND EFFECTIVE VISCOSITY FOR FLOW SIMULATIONS IN DUAL-SCALE FIBROUS POROUS MEDIA

Jingang Lu and Wook Ryol Hwang*

School of Mechanical Engineering, Research Centre for Aircraft Parts Technology (ReCAPT),
Gyeongsang National University, Jinju, 52828, Korea

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ABSTRACT

In this work, the Stokes-Brinkman model has been applied to investigate a flow in a channel coupling with a flow in a fibrous porous medium. According to our previous work [J. Lu et al., Composites part A, vol. 100, p. 9-19], the anisotropic slip coefficient at the interface between the channel flow and the fibrous porous medium has been characterized using the flow rate matching method. The effective viscosity in the Brinkman equation, which should also be anisotropic, was found to be as a function of the slip coefficient. By accurately characterizing the permeability and the effective viscosity, the Brinkman model with a pure domain can be applied to replace the actual fibrous porous medium, with which the computing cost will be expected to reduce considerably. An example problem has been solved numerically to validate the accuracy of the Brinkman model with such a series of non-identity effective viscosity ratios, it was found that, the averaged velocity at the interface from the actual flow agrees extremely well with the velocity at the interface using the Stokes-Brinkman model, so as to the Darcy velocity far away from the interface. Within the very narrow boundary layer adjacent to the interface, there exists a non-ignorable difference between the actual flow and the one from the Brinkman model, which may implies that the stress continuity boundary condition at the interface has its limitation in expressing the flow in the boundary layer.

1 INTRODUCTION

Darcy's law (e.g. $u_D = -K/\mu \nabla p$), was widely accepted and applied to describe the flow through a porous medium since 1850s. However, when there is a flow over the porous medium, the velocity at the fluid/porous interface has been proved to be several orders of magnitude greater than the Darcy velocity u_D, which implies the existence of the thin boundary layer adjacent to the interface. Since there appears no macroscopic shear term included, the Darcy equation is not compatible with the existence of a boundary layer region in the porous medium. Thus to avoid this fundamental limitation of the Darcy equation, Beavers and Joseph [1] proposed a mathematic assumption to force the velocity at the interface scaling with the velocity gradient (e.g. $u - u_D = \beta \partial u/\partial n$, with β being slip coefficient), which has been validated by their own experimental results. In fact, Beavers and Joseph simply fix the interfacial slip to a certain value, neglecting the existence of the boundary layer. In other words, according to Beavers and Joseph, there appears a jump of the tangential velocity component across the interface, which is obviously impermissible. On the other hand, in order to account for the transitional flow between boundaries, Brinkman [2] extended the traditional form of Darcy's law by adding the Brinkman term (e.g. $\mu d^2u/dy^2 - \mu/Ku = dp/dx$, with μ being effective viscosity). It is believed that Brinkman's extension of Darcy's law is mathematically and physically preferable to Darcy's law when examining the boundary layer effects adjacent to the interface in porous medium [3,4]. Neale [5] analytically solved a channel flow coupled with a flow through the porous media. The Stokes equation and the Brinkman equation were used to describe the channel flow and the porous flow, respectively. At the fluid/porous interface, the velocity continuity and the stress continuity boundary condition have been applied. Comparison between the theoretical solution at the interface and Beavers and Joseph's assumption showed that the effective viscosity ratio is identical to squares of the slip coefficient by Beavers and Joseph (e.g. $\mu/\mu = \alpha_{BJ}{}^{2}$). Even though the effective viscosity ratio is customarily regarded as identity, in our previous work [6], using the flow rate matching method, we have provided the value of the slip coefficient α_{BJ} as a function of fibre volume fraction in the fibrous porous media, which implies that the effective viscosity is no longer identical to the fluid viscosity, but a function of the fibre volume fraction and the porous structure. In addition, the effective viscosity should be anisotropic because of the anisotropic property of the slip coefficient.

In the present work, the Stokes-Brinkman model has been applied to investigate a flow in a channel coupling with a flow in a fibrous porous medium. By accurately characterizing the permeability and the effective viscosity, the Brinkman model with a pure domain can be applied to replace the actual fibrous porous medium, with which the computing cost will be expected to reduce considerably. An example problem has been solved numerically to validate the accuracy of the Brinkman model with such a series of non-identity effective viscosity ratios, it was found that, the averaged velocity at the interface from the actual flow agrees extremely well with the velocity at the interface using the Stokes-Brinkman model, so as to the Darcy velocity far away from the interface. Within the very narrow boundary layer adjacent to the interface, there exists a non-ignorable difference between the actual flow and the one from the Brinkman model, which may imply that the stress continuity boundary condition at the interface has its limitation in expressing the flow in the boundary layer.

2 STOKES-BRINKMAN MODEL

In this section, we will briefly present how to derive the solution of the Stokes-Brinkman model in each domain based on some certain boundary conditions. One can also find the process from Neal's work [5].

As shown in Figure 1, the flow in the channel $(0 < y < H)$ is controlled by the Stokes equation:

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x}, \tag{1}$$

where μ is the viscosity of the fluid, p is the pressure, u is the fluid velocity in the channel. As to the flow in the porous medium $(-\infty < y \le 0)$, it is controlled by the Brinkman equation, which can be written as follows:

$$\mu\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{K}u = \frac{\partial p}{\partial x}, \tag{2}$$

with the symbol μ being the effective viscosity, u is the fluid velocity in the porous medium. At the top of the channel wall, there is a no-slip boundary condition, e.g. $u = 0,\ y = H$. At the region in the porous medium, where the position is far away from the interface, the effect of Brinkman equation reduces to Darcy's law, e.g. $u = -(K/\mu)\nabla p = u_D,\ y \to -\infty$. At the fluid/porous interface, there appear velocity continuity and stress continuity, which can be expressed as:

$$u = u,\ \mu\frac{\partial u}{\partial y} = \mu\frac{\partial u}{\partial y},\ y = 0. \tag{3}$$

According to the governing equations and boundary conditions above, the solution in the fluid channel is:

$$u = u_D\left(\frac{\sigma\left(\sigma + 2\sqrt{\mu/\mu}\right)}{2\left(1+\sigma\sqrt{\mu/\mu}\right)} + \frac{\left(\sigma^2 - 2\right)\sqrt{\mu/\mu}}{2\left(1+\sigma\sqrt{\mu/\mu}\right)}\left(\frac{y}{\sqrt{K}}\right) - \frac{1}{2}\left(\frac{y}{\sqrt{K}}\right)^2\right), \tag{4}$$

where $\sigma = H/\sqrt{K}$. The solution in the porous medium can be written as:

$$u = u_D\left(1 + \frac{\sigma^2 - 2}{2\left(1+\sqrt{\mu/\mu}\right)} e^{\sqrt{\frac{\mu}{\mu}}\frac{y}{\sqrt{K}}}\right). \tag{5}$$

Following the no-slip boundary condition at the top of the channel wall and the slip boundary condition at the interface proposed by Beavers and Joseph that $u - u_D = \left(\alpha/\sqrt{K}\right)\partial u/\partial n$, $\mathrm{y} = 0$, one can also get the solution in the channel:

$$u = u_D\left(\frac{\sigma(\sigma+2\alpha)}{2(1+\sigma\alpha)} + \frac{\left(\sigma^2-2\right)\alpha}{2(1+\sigma\alpha)}\left(\frac{y}{\sqrt{K}}\right) - \frac{1}{2}\left(\frac{y}{\sqrt{K}}\right)^2\right). \tag{6}$$

Comparing Eq. (4) and Eq. (6), one can readily obtain the relation between the effective viscosity in the Brinkman equation and the dimensionless slip coefficient by Beavers and Joseph such that:

$$\mu = \mu {\alpha_{BJ}}^2. \tag{7}$$

In our previous work, the dimensionless slip coefficients α_{BJ} for various packing structures (e.g. Quad, Hex1 and Hex2), in both transverse and longitudinal directions have been accurately characterized, which implies that one can readily characterized the anisotropic effective viscosity based on Eq. (7).

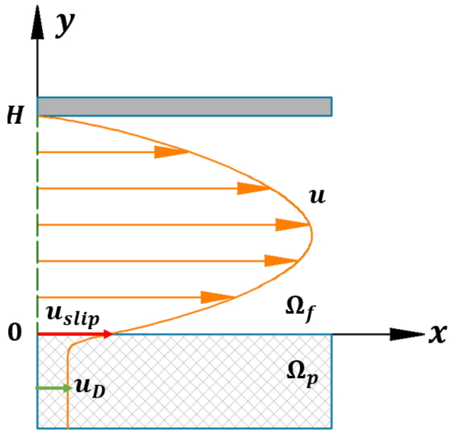


Figure 1: A schematic description of a flow in the channel coupled with a flow in the porous medium

3 RESULTS AND DISCUSSIONS

In this section, we will first present the effective viscosity in the fibrous porous media for three different packing structures (e.g. Quad, Hex1 and Hex2) in both transverse direction and longitudinal direction. Then we introduce an example problem to validate the accuracy of the Brinkman model with such a series of non-identity effective viscosity ratios.

3.1 The anisotropic effective viscosity in fibrous porous media

According to our previous work on characterizing the anisotropic slip coefficient for the flow over the fibrous porous media, the expression of the slip coefficient can be expressed as:

$$\beta_\theta = \frac{4 - Q_\theta^*}{\left(Q_\theta^* - 1\right)H - 6K_\theta/H} \quad (\theta = \| \text{ or } \perp), \tag{8}$$

with Q_θ^* being the flow rate ratio, " $\|$ " denotes longitudinal direction and " $\perp$ " is transverse direction. Combing Eq. (7) and (8) and the relation that $\beta = \alpha/\sqrt{K}$, one can readily get the value of the effective viscosity in the Brinkman equation. Plotted in Figure 2 is the effective viscosity ratio μ/μ as a function of the fiber volume fractions for different packing structures in both longitudinal and transverse directions. It is shown that the effective viscosity ratios decrease as the fiber volume fraction increases for all the three different packing structures in both longitudinal and transverse directions. For various packing structures, the effective viscosity in the transverse direction is larger than the one in the longitudinal direction when the fibers are sparsely packed, and the opponent results appear in the densely packed region. Interestingly, in the longitudinal direction, for the same fiber volume fraction, the Quad structure always gets the largest value of the effective viscosity ratio, and the Hex1 structure gets the least value. However, in the transverse direction, even though the Hex1 structure always gets the least value of the effective viscosity ratio, in the sparsely packed region, the Quad structure gets a larger effective viscosity value than the Hex2 structure, when the fibers are densely packed, the Hex2 structure gets a larger effective viscosity value.

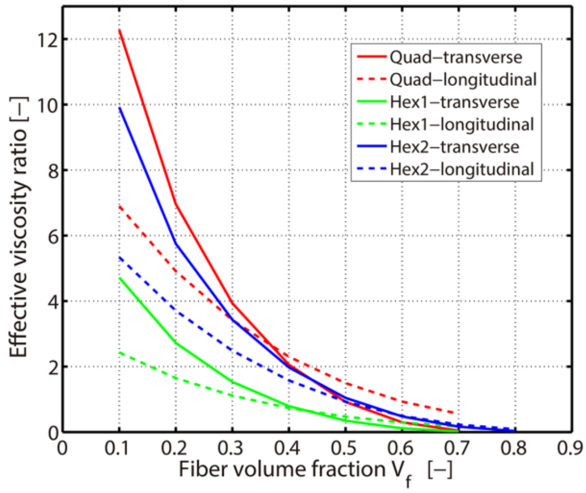


Figure 2: The effective viscosity ratio μ/μ as a function of the fiber volume fractions for different packing structures and in both longitudinal and transverse directions

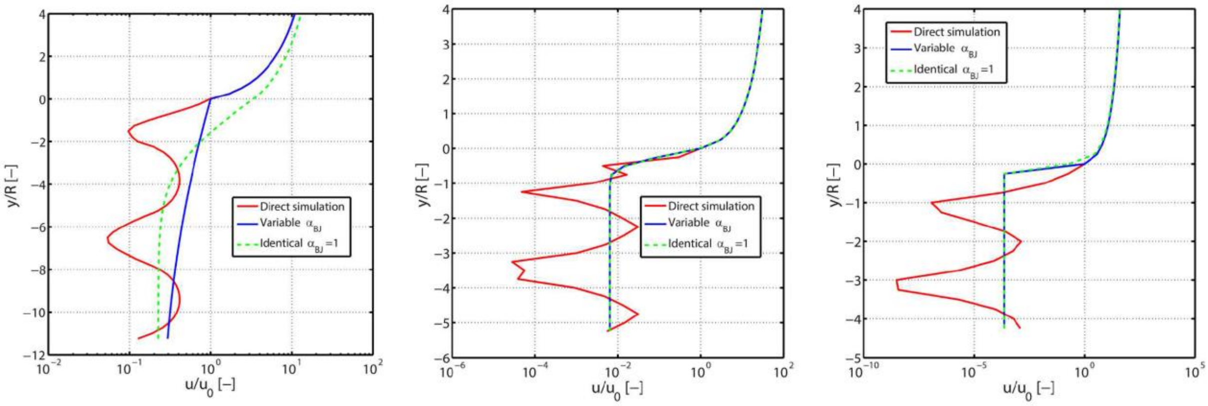
3.2 An example problem

Now we start to solve one example problem such that a 2D channel flow coupling with a flow through the fibrous porous medium. The packing structure is chosen as Quad. The channel height ratio is set as 32 (e.g. $H/R=32$). For different fibre volume fractions, the permeability and the effective viscosity ratio are firstly characterized, then we substitute these values into either Eq. (4) or (5), assuming y being zero to theoretically calculate the velocity at the interface. At the same time, a 2D actual flow over the fibrous porous media was solved using COMSOL Multiphysics 4.4 with the quadratic velocity and the linear pressure interpolations. In addition, fluid inertia is neglected due to small scale of the flow problem. The averaged velocity at the interface from the simulation was taken out to compare with the analytical one. The results are listed in the Table 1. It shows that the averaged slip velocity at the interface from the actual flow agrees extremely well with the corresponding theoretical one among all the range of the fibre volume fractions.

Figure 3 shows the comparison of velocity distribution between actual flow and the Stokes-Brinkman model with identity and non-identity effective viscosity ratios for different fiber volume fractions. One can observe that for the channel flow ($y/R>0$), the actual flow agrees pretty well with the theoretical Stoke-Brinkman model using a non-identity effective viscosity ratio. At the interface, the slip velocity from the analytical Stokes-Brinkman model with an identity effective viscosity $\left(\mu/\mu\equiv1\right)$ decreases as the fiber volume fraction increases. When the fiber volume fraction is 0.5 or the effective viscosity ratio is close to identity, the velocity field in both channel and the porous media agree extremely well with the actual flow. According to the trend of the velocity profile in Figure 3, far away from the interface, the velocity falls to Darcy velocity for both identity and non-identity effective viscosity ratio. Among the boundary layer adjacent to the interface, there shows a nonnegligible difference between the actual flow and the theoretical solution from Stokes-Brinkman model, which implies that the stress continuity boundary condition at the interface in Stokes-Brinkman model has its limitation in expressing the flow in the boundary layer.

V_f	μ/μ	u_s^{Actual}	$u_s^{Analytical}$	Relative Error
0.1	17.0	1.74e-7	1.78e-7	0.021
0.2	7.66	2.18e-7	2.21e-7	0.014
0.3	4.04	2.48e-7	2.51e-7	0.011
0.4	2.05	2.71e-7	2.73e-7	0.009
0.5	0.90	2.89e-7	2.92e-7	0.008
0.6	0.29	3.05e-7	3.07e-7	0.007
0.7	0.04	3.18e-7	3.20e-7	0.006

Table 1: Validation of the effective viscosity based on the slip velocity at the interface



(a) (b) (c)

Figure 3: Comparison of velocity distribution between actual flow and the Stokes-Brinkman model with identity and non-identity effective viscosity ratio. (a) $V_f = 0.1$, (b) $V_f = 0.5$, (c) $V_f = 0.7$

4 CONCLUSIONS

In this work, the Stokes-Brinkman model has been applied to investigate a flow in a channel coupling with a flow in a fibrous porous medium. By accurately characterizing the permeability and the effective viscosity, the Brinkman model with a pure domain can be applied to replace the actual fibrous porous medium, with which the computing cost will be expected to reduce considerably. An example problem has been solved numerically to validate the accuracy of the Brinkman model with a series of non-identity effective viscosity ratios, it was found that, the averaged velocity at the interface from the actual flow agrees extremely well with the velocity at the interface using the Stokes-Brinkman model, so as to the Darcy velocity far away from the interface. Within the very narrow boundary layer adjacent to the interface, there exists a non-ignorable difference between the actual flow and the one from the Brinkman model, which may imply that the stress continuity boundary condition at the interface has its limitation in expressing the flow in the boundary layer.

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REFERENCES

[1] G.S. Beavers and D.D. Joseph, Boundary conditions at a naturally permeable wall. *Journal of Fluid Mechanics*, **30** (01), 1967, pp. 197-207.

[2] H. Brinkman, A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. *Applied Scientific Research,* **1** (1), 1949, pp. 27-34.

[3] P.G. Saffman, On the boundary condition at the surface of a porous medium. *Studies in Applied Mathematics,* **50** (2), 1971, pp. 93-101.

[4] T.S. Lundgren, Slow flow through stationary random beds and suspensions of spheres. *Journal of Fluid Mechanics,* **51** (02), 1972, pp. 273-299.

[5] G. Neale and W. Nader, Practical significance of Brinkman's extension of Darcy's law: coupled parallel flows within a channel and a bounding porous medium. *The Canadian Journal of Chemical Engineering,* **52** (4), 1974, pp. 475-478.

[6] J.G. Lu, H.K. Jang S. B. Lee and W.R. Hwang, Characterization on the anisotropic slip for flows over unidirectional fibrous porous media for advanced composites manufacturing, *Composites Part A*, **100**, 2017, pp.9-19.