A coupled FEM/PD based on substructure method for progressive damage analysis of composite laminate

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Abstract: A new method of coupling finite element method (FEM) and peridynamics (PD) is proposed to analyze the progressive damage of fiber reinforced composite laminate. The computational efficiency of FEM is higher than peridynamics. But peridynamics is good at calculating discontinuous solution. In order to exert the advantages of the above two methods, the solution is divided into three parts. In the dangerous region, peridynamics is adopted. And FEM is used in the non-dangerous region. The remaining region is the coupling FEM/PD region, where the peridynamics particles are interacted with neighboring finite element nodes each other. And the contribution stiffness of FEM and peridynamics particles can be written in the global stiffness matrix together. The substructure method is also used to reduce computational effort further. The progressive damage and failure modes of composite laminates under a tensile load can be successfully analyzed. It is shown that this proposed method had a great potential to analyze the mechanical behavior of composites.

Keywords: composite laminate; peridynamics; coupling; damage; failure mode

1. Introduction

With the advantages of high strength, high modulus, and low density, composites have been widely used in aerospace, mechanical engineering, civil engineering, and other engineering fields in recent years. However, due to the complexity of composite, its failure prediction is a very challenging task [1-3].

The traditional theory has difficulty in solving discontinuous problems, such as crack, damage, fracture, etc. Silling [4] introduced the peridynamics theory. Different from the traditional continuum theory, the peridynamics is to establish motion equation based on the domain integral. This characteristic of peridynamics shows that the initiation and propagation of damage at any position in material is spontaneous and may appear along any path without following any crack propagation rules. For peridynamics, the internal force of the continuum is expressed by the non-local interaction between any two material points in the region, and the damage is part of the constitutive model. When the load reaches a certain value, the damage will initiate and propagate by using a criterion.

As an emerging theory, peridynamics has attracted more and more attention. In recent years, many experts and scholars have researched on it [5-8] and have made a lot of research results. Based on peridynamics, Huang [9] *et al.* simulated the brittle damage of concrete and carried out impact failure simulation and tensile crack propagation simulation, which validated peridynamics in the simulation of brittle material damage and failure to a certain extent. Due to the lower solving efficiency, peridynamics is obviously not as fast and convenient as finite element method in solving continuous problems. Therefore, in order to exert the advantages of

peridynamics and finite element method respectively, several scholars researched a lot on the combination of bonded-based peridynamics and FEM [9, 10].

At present, there is no literature coupling state-based peridynamics and FEM. In this paper, the state-based peridynamics and FEM were coupled to solve the crack propagation problems of fiber reinforced composites.

2. Coupling Calculation Scheme

2.1 peridynamics

Based on the state-based peridynamics, the motion equation of an arbitrary point $\boldsymbol{x}$ at an arbitrary moment t can be written as

$$\rho_k \ddot{\boldsymbol{u}}_k = \sum_{j=1}^{N} \left(\mathbf{t}_{kj}(\boldsymbol{u}_j - \boldsymbol{u}_k, \boldsymbol{x}_j' - \boldsymbol{x}_k, t) - \mathbf{t}_{jk}(\boldsymbol{u}_k - \boldsymbol{u}_j, \boldsymbol{x}_k' - \boldsymbol{x}_j, t) \right) V_j + \boldsymbol{b}_k(\boldsymbol{x}, t) \tag{2}$$

For quasi-static problems, Eq. (2) can be written as

$$\mathbf{0} = \sum_{j=1}^{N} \left(\mathbf{t}_{kj}(\boldsymbol{u}_j - \boldsymbol{u}_k, \boldsymbol{x}_j' - \boldsymbol{x}_k) - \mathbf{t}_{jk}(\boldsymbol{u}_k - \boldsymbol{u}_j, \boldsymbol{x}_k' - \boldsymbol{x}_j) \right) V_j + \boldsymbol{b}_k(\boldsymbol{x}) \tag{3}$$

where $\mathbf{t}_{kj}$ is the force density vector for the material point at location $\boldsymbol{x}_j$ exerting on the material point at location $\boldsymbol{x}_k$, which can be expressed as

$$\mathbf{t}_{kj}(\boldsymbol{u}_j - \boldsymbol{u}_k, \boldsymbol{x}_j' - \boldsymbol{x}_k) = \frac{1}{2} A_{kj} \frac{\boldsymbol{y}_j - \boldsymbol{y}_k}{|\boldsymbol{y}_j - \boldsymbol{y}_k|} \tag{4}$$

And

$$\mathbf{t}_{jk}(\boldsymbol{u}_k - \boldsymbol{u}_j, \boldsymbol{x}_k' - \boldsymbol{x}_j) = -\frac{1}{2} B_{jk} \frac{\boldsymbol{y}_j - \boldsymbol{y}_k}{|\boldsymbol{y}_j - \boldsymbol{y}_k|} \tag{5}$$

where A_{kj} and B_{jk} are auxiliary parameters.

$$A_{kj} = \frac{4\delta}{|\boldsymbol{x}_j - \boldsymbol{x}_k|} da\theta_k + 4\delta\left(\mu_F b_F + b_{FT} + \mu_T b_T\right) s_{kj} \tag{6}$$

$$B_{jk} = \frac{4\delta}{|\boldsymbol{x}_j - \boldsymbol{x}_k|} da\theta_j + 4\delta\left(\mu_F b_F + b_{FT} + \mu_T b_T\right) s_{kj} \tag{7}$$

Substituting Eq. (4), (5), (6), and (7) into Eq. (3)

$$\mathbf{0} = \sum_{j=1}^{N} \left(4\delta\left(\mu_F b_F + b_{FT} + \mu_T b_T\right) s_{kj} + \frac{2ad\delta}{|\boldsymbol{x}_j - \boldsymbol{x}_k|}\left(\theta_k + \theta_j\right) \right) V_j \frac{\boldsymbol{y}_j - \boldsymbol{y}_k}{|\boldsymbol{y}_j - \boldsymbol{y}_k|} + \boldsymbol{b}_k(\boldsymbol{x}) \tag{8}$$

According to Ref. [20], the interaction between $\boldsymbol{x}_k$ and $\boldsymbol{x}_j$ is written in the form of spring stiffness matrix, then

$$\mathrm{K}_{kj} = \frac{4\delta\left(\mu_F b_F + b_{FT} + \mu_T b_T\right)}{\left|\boldsymbol{x}_j - \boldsymbol{x}_k\right|} V_k V_j \begin{bmatrix} \xi_1^2 & \xi_1\xi_2 & -\xi_1^2 & -\xi_1\xi_2 \\ \xi_1\xi_2 & \xi_2^2 & -\xi_1\xi_2 & -\xi_2^2 \\ -\xi_1^2 & -\xi_1\xi_2 & \xi_1^2 & \xi_1\xi_2 \\ -\xi_1\xi_2 & -\xi_2^2 & \xi_1\xi_2 & \xi_2^2 \end{bmatrix} \tag{9}$$

$$p_k = \frac{2ad\delta}{\left|\boldsymbol{x}_j - \boldsymbol{x}_k\right|}\left(\theta_k + \theta_j\right) V_j \frac{\boldsymbol{x}_j - \boldsymbol{x}_k}{\left|\boldsymbol{x}_j - \boldsymbol{x}_k\right|} + b_k \tag{10}$$

where K_{kj} is the element stiffness matrix, and p_k is the right-hand-side column vector.

To sum up, by writing the state-based peridynamics in the FEM form and assembling peridynamics region and FEM region into a global stiffness matrix, the coupling scheme can be conducted in the finite element form, and further reduced the amount of calculation through Guyan reduction.

The strength of fiber reinforced composite will be different in different directions. The specific fracture criterion is shown in Ref. [11].

2.2 Coupling Scheme

The coupling region is shown in Fig. 1, where the blue points and green blocks are peridynamics particles and finite element nodes, respectively. When $\delta = 3\Delta x$, each peridynamics particle interacts with 28 particles around it. For the peridynamics particles at the edge of peridynamics region, the active region includes 17 peridynamics particles and 11 finite element nodes. Fig. 2 shows the interaction between the finite element node *k* in the coupling region and its surrounding nodes. In its active region, the left three blue points are peridynamics particles, and the right five green blocks are the finite element nodes. Node *k* interacts with all nodes in its active region in finite element mode.

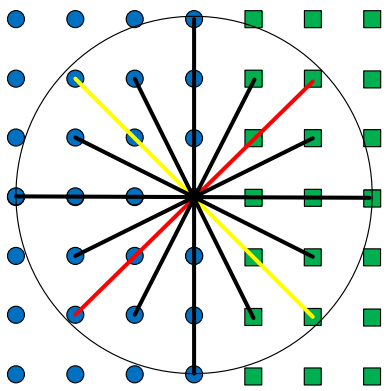


Fig.1 Interaction between PD particles and particles within their region in the coupling region

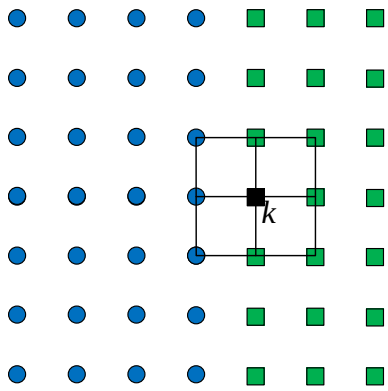


Fig.2 Interaction between FEM nodes and particles within their region in the coupling region

2.3 Guyan Reduction

The coupled equation is written in the block form [12]

$$\begin{bmatrix} k_{ii} & k_{io} \\ k_{oi} & k_{oo} \end{bmatrix} \begin{bmatrix} u_i \\ u_o \end{bmatrix} = \begin{bmatrix} P_i \\ P_o \end{bmatrix} \tag{11}$$

where the subscript i is the dependent degree of freedom (DOF), the subscript o is the master DOF. Using Guyan reduction, Eq. (11) can be written as

$$k_{c0}u_o = P_{c0} \tag{12}$$

Eq. (12) is the finite element equation after reduction, and the order of k_{c0} is the same as that of k_{oo}, where

$$k_{c0} = k_{oo} - k_{oi}k_{ii}^{-1}k_{io} \tag{13}$$

$$P_{c0} = P_o - k_{ii}^{-1}k_{io}P_i \tag{14}$$

The displacement value of the dependent DOF is restored by Eq. (15)

$$u_i = -k_{ii}^{-1}k_{io}u_o \tag{15}$$

3. Numerical Examples

In this section, the proposed method is applied to study composite with an initial crack. The geometrical dimension and boundary conditions are shown in Fig. 3, with length $a = 76.2\text{mm}$ and width $b = 152.4\text{mm}$. The center of composite has an initial crack with length $c = 17.78\text{mm}$. θ is the angle between the fiber direction and the horizontal direction. The simple boundary condition is applied on the lower edge of the plate, and the displacement boundary condition is applied on the upper edge. The material parameters and strength of composite are shown in Tab. 1.

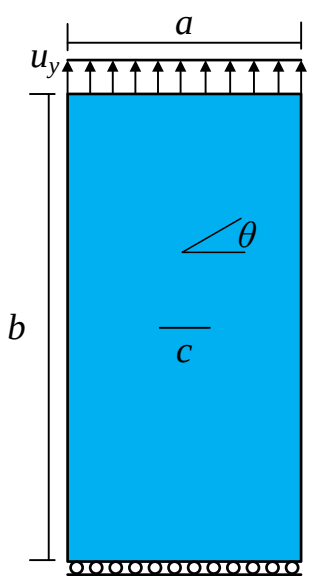


Fig.3 Model of composite with crack

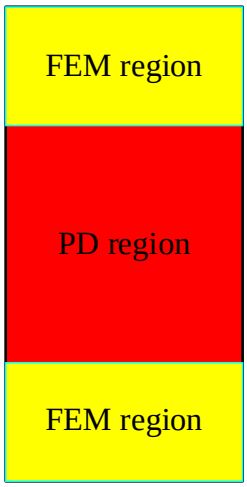


Fig.4 Region partition of composite

Tab. 1 Material parameters and strength

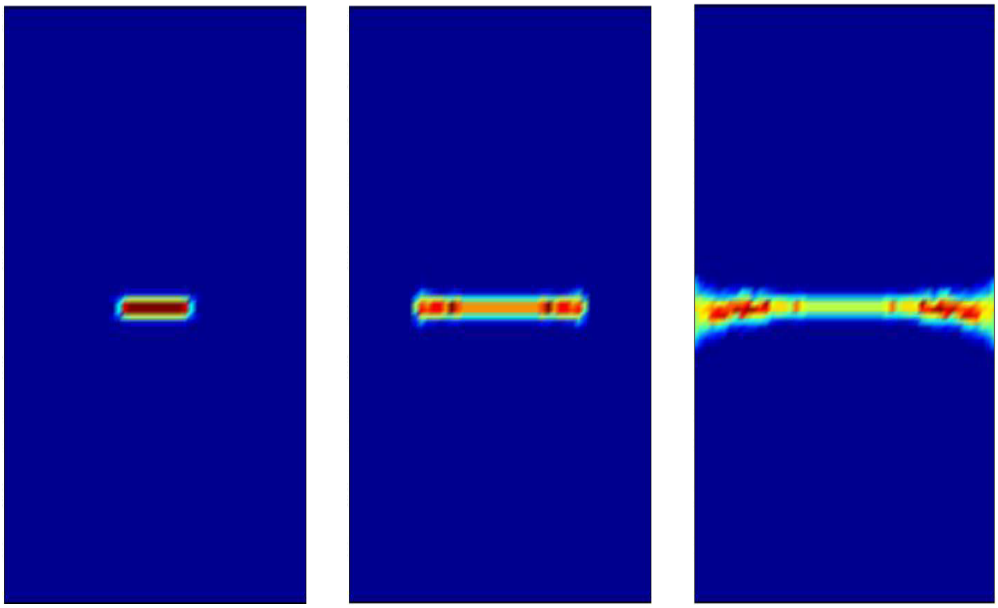
E_{11}	E_{22}	G_{12}	ν_{12}	X^T	X^C	Y^T	Y^C
23.2MPa	1.3MPa	0.9MPa	0.28	0.412 MPa	0.225 MPa	0.00872 MPa	0.0243 MPa

The division of PD region and FEM region is shown in Fig. 4, in which the yellow part is FEM region, and the red part is PD region. The specific division rules are shown as follows

$$\begin{cases} \text{FEM region} & y < 30\text{mm or } y > 121\text{mm} \\ \text{PD region} & 30\text{mm} \le y \le 121\text{mm} \end{cases}$$

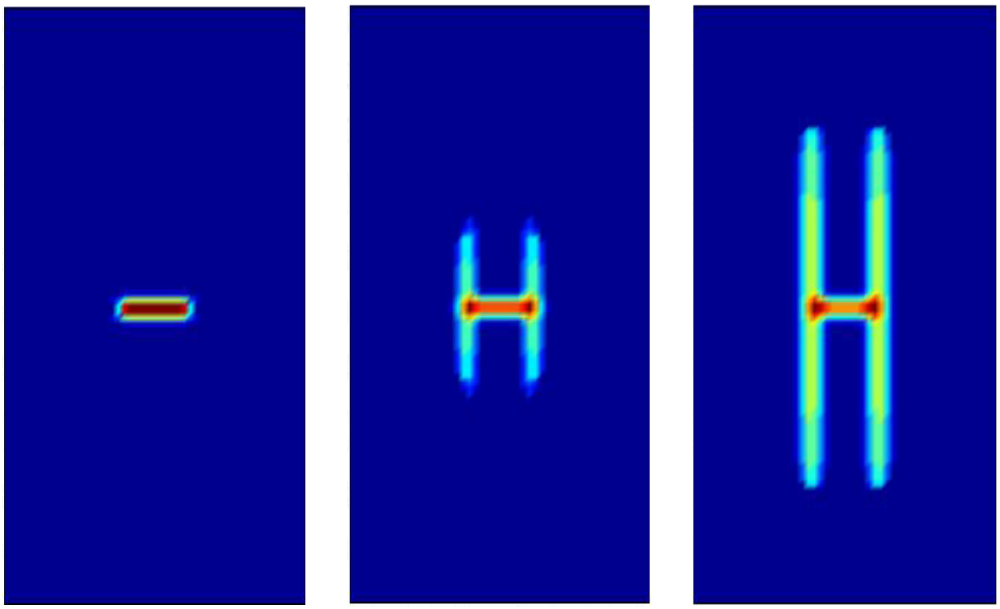
where y is the ordinate.

Then, the proposed method was used to simulate the crack propagation process of composite. First, an initial displacement u_0 is selected to calculate the displacement field. Then, any bond damage in the structure according to the fracture criterion is checked. If damage is not happened, a pre-given displacement increment Δu is added and the displacement field is recalculated. If damage has been happened, the structural stiffness and the displacement field under the same displacement boundary conditions are required to be recalculated. Repeat the single-layer plate is damaged entirety after repeating the above steps.



(a)step 1 (b)step 3030 (c)step 3040

Fig.5 Crack propagation process of composite with $\theta=0^0$



(a)step 1 (b)step 1300 (c)step 2616

Fig.6 Crack propagation process of composite with $\theta=90^0$

As shown in Fig. 5, for $\theta=0^0$, the cracks propagated horizontally in two directions until the through cracks appeared. For $\theta=90^0$, the cracks propagated vertically in two directions until reaching the edge of PD region as shown in Fig. 6.

4. Conclusion

This paper coupled the state-based PD theory and FEM to solve the fracture problems of the fiber reinforced composites under the quasi-static condition, assembled the state-based PD theory and FEM into a global stiffness matrix, and reduced the computational load through Guyan reduction. The numerical examples show that this coupling method has high efficiency and accuracy. The proposed method provides a new idea for the coupling of PD theory and FEM method in calculating crack propagation problems.

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