# SENSITIVITY OF ORTHOTROPIC MATERIAL PARAMETERS USED WITHIN THE STRAIN INVARIANT FAILURE THEORY

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**Keywords:** Composite specimens, Matrix failure, Onset theory, SIFT, Material properties

## ABSTRACT

Understanding failure within composite materials has been an ongoing area of research over the last several decades. The difficulty in understanding their properties is fundamentally due to the non-homogeneous nature of the microstructure, which is made up of two materials with distinctly different properties. Many newly emerging failure theories to acknowledge this non-homogeneity using Representative Volume Elements (RVEs), where the properties of the fibre and matrix constituents are retained throughout the modelling process. However, this improvement made in the analysis procedure is complicated by the requirement to obtain a full set of orthotropic material properties. The fibre and matrix constituents are often assumed to be isotropic on their own, but this is not the case. In this paper the authors examine a physics based failure criterion; the Strain Invariant Failure Theory (SIFT) and its sensitivity to various anisotropic material properties. It was found that assuming the fibre to be isotropic was responsible for 55% difference in the failure criterions' prediction. Given this significant difference, additional examination of constituent properties and their effects on these dehegemonised failure theories was looked at.

## 1 INTRODUCTION

As to now there is still no widely accepted definition as to when a composite material should be termed to have failed. This is because composite materials are made up of two main constituents; the fibre and the matrix, which often results in a complex progression of damage. This implies that when a composite material ultimately fails; it is a build-up of micromechanical failure events at the matrix and fibre level. From a structural level this has huge implications as the aerospace industry is constantly under pressure to minimise costs with air travel becoming a necessity for global business. However, with the industry being highly regulated and materials being used to form primary structures, there is a need for composites to be characterised accurately and appropriate failure criteria need to be identified. Without these, large margins of safety are applied to a structural design to account for the uncertainties with failure prediction. This counteracts a lot of the benefits in strength and weight saving that the industry could potentially see.

There have been numerous failure theories proposed for materials over the past century [1]. However, their acceptance is often determined by a balance between accuracy and their ease of use. The Strain Invariant Failure Theory (SIFT) [2-4] (also known as Onset Theory), proposed by Gosse et al, offers a novel approach to predicting composite mechanical properties.

SIFT, unlike other failure theories, is physics-based, demanding physical consistency across all length scales. The physics basis of SIFT ensures that it will be ultimately generalizable to any composite system. In order to apply any failure theory to real composite design, tremendous fundamental research is required to accurately observe, interpret and model the extremely complex internal state of composite structures. To simplify the modelling procedure, assumptions regarding material properties are often made.

In this paper, a sensitivity analysis will be conducted to determine which material properties are critical for RVE modelling. SIFT is the failure theory that will be used to process the biaxial tension results published in literature [5-9]. SIFT proposes that once εdil reaches its critical value, the matrix constituent fails due to an excessive increase in its volumetric shape, i.e. failure is dilatationally driven. A plot of the truncated plane given by the First Strain Invariant (or εdil) in principal strain space is shown in Figure 1, where: εdil is the dilatational strain invariant.

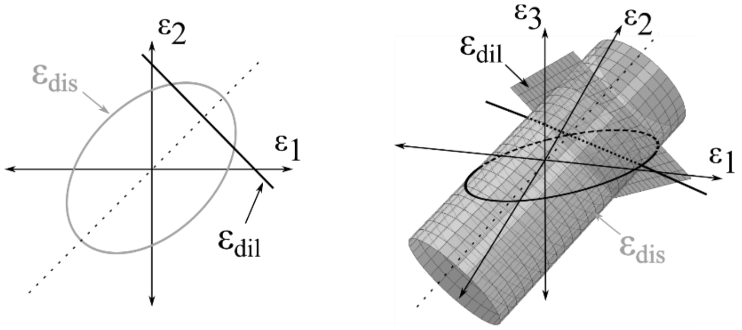


Figure 1: Different views of the tension octant given by the Strain Invariant Failure theory [5].

## 2. METHODOLOGY

This investigation performs a parametric study on the influence of material parameters on the overall failure prediction given by Onset Theory. Onset Theory requires a full set of material properties for the fibre and the matrix. The theory was proposed with the intention that it can easily be adopted by users as the process of implementing the criterion is relatively simple without the need for any specific calibration parameters. The theory requires thirteen material properties for the fibre and for the matrix as shown in Table 1. It is known that some of these properties are very difficult to characterise as they require unconventional measuring techniques which most researchers do not have access to. For this reason, a lot of assumptions are usually made in the modelling process. However, it is important to note that these assumptions can play an important role in the final failure predictions.

| Property | Required for: |
|---|---|
| Volume Fraction | F, M |
| $E_1$ (GPa) | F, M |
| $E_2$ (GPa) | F, M |
| $E_3$ (GPa) | F, M |
| $G_{12}$ (GPa) | F, M |
| $G_{23}$ (GPa) | F, M |
| $G_{13}$ (GPa) | F, M |
| $\nu_{12}$ | F, M |
| $\nu_{23}$ | F, M |
| $\nu_{13}$ | F, M |
| $\alpha_1$ (/$^o$C) | F, M |
| $\alpha_2$ (/$^o$C) | F, M |
| $\alpha_3$ (/$^o$C) | F, M |

Table 1: Material properties required for the Fibre (F) and the Matrix (M).

Macro-mechanical level (lamina level) strains previously published for a range of tests [5, 9] have been used in this paper as inputs for running of the models. Only a subset of this data was reprocessed for the purposes of this paper. Table 2, shows some of the macroscopic level failure strains obtained for the biaxial tension tests. All the data presented in the previous investigation was found to be well captured by the plane given by the First Strain Invariant ($\varepsilon_{dil}$). The investigation is broken down into three main areas:

1. Isotropic fibre and matrix,
2. Anisotropic fibre and isotropic matrix.
3. Sensitivity of crucial anisotropic fibre properties.

| Specimen Nº: | Fibre angle: (degrees) | Lamina failure strains: | | | | | |
|---|---|---|---|---|---|---|---|
| | | $\varepsilon_{11}$ (µ-strain) | $\varepsilon_{22}$ (µ-strain) | $\varepsilon_{33}$ (µ-strain) | $\gamma_{12}$ (µ-strain) | $\gamma_{13}$ (µ-strain) | $\gamma_{23}$ (µ-strain) |
| 1 | 90 | -200 | 14513 | -6530 | -0.0000319 | 1.69E-07 | 1.68E-07 |
| 2 | | -190 | 13387 | -6020 | -0.0000295 | 1.56E-07 | 1.55E-07 |
| 3 | | -170 | 11886 | -5350 | -0.0000262 | 1.38E-07 | 1.37E-07 |
| 4 | | -140 | 9884 | -4450 | -0.0000218 | 1.15E-07 | 1.14E-07 |
| 5 | | -160 | 11260 | -5070 | -0.0000248 | 1.31E-07 | 1.3E-07 |

Table 2: Macroscopic failure strains from biaxial tension tests [5, 9].

This investigation starts by simplifying the overall investigation by assuming the fibre and the matrix are both isotropic in nature. It is an assumption that is quite commonly made in literature due to its conveniences. This assumption simplifies the number of properties listed in Table 1, from thirteen material properties down to five values (shown in Table 3). The properties for EP280 Prepreg which is the material considered in this investigation are listed in Table 4.

| Property | Required for: |
|---|---|
| Volume Fraction | F,M |
| $E_1 = E_2 = E_3$ (GPa) | F,M |
| $G_{12} = G_{13} = G_{23}$ (GPa) | F,M |
| $\nu_{12} = \nu_{13} = \nu_{23}$ | F,M |
| $\alpha_1 = \alpha_2 = \alpha_3$ (/ºC) | F,M |

Table 3: Material properties required for the Fibre (F) and the Matrix (M).

| Property | Fibre | Matrix |
|---|---|---|
| Volume Fraction | 0.5 | 0.5 |
| E (GPa) | 259 | 3.15 |
| G (GPa) | 20 | 1.21 |
| $\nu$ | 0.3 | 0.3 |
| $\alpha$ (/ºC) | 0 | $2.38\times10^{-5}$ |

Table 4: EP280 neat resin and Grafil-34-700 isotropic material properties

Using these properties, the SIFT methodology was followed where the macroscopic strains within the failed specimen [10, 11] where amplified using Micro-Mechanical Enhancement factors (MMEs) based on the properties shown in Table 3. The concept of MMEs was established from the idea that if a cuboid of neat resin had a strain applied to it, then the effect of embedding a stiffer fibre within that cuboid would result in an added strain field around the resin due to the fibre behaving as a stress concentrator on the surrounding matrix. This concept is analysed using a Representative Volume Element (RVE) of the material at a micro-mechanical level. The SIFT approach is quite simplistic from a postprocessing view as once the MMEs are obtained from the material system, it is a matter of applying Eqn. 1 to obtain the micro-mechanical level failure strains (εm). Where $M^m$ is the MME matrix, ε ̅is the macroscopic strains, Am is the strain induced per unit of temperature change where ΔT is the change from cureing to test temperature. Eqn. 2 is described by SIFT as the Dilatational invariant (εdil) or (J1). The subscripts represent the tensor directions associated with the material system. Critical location for the dilatational failure strains were identified from the FEA contour plots.

$$\varepsilon_i^m = M_{ij}^m \bar{\varepsilon}_j + A_i^m \Delta T \tag{1}$$

$$\varepsilon_{\mathrm{dil}} \equiv J_1 = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{2}$$

## 3. RESULTS

### 3.1 Isotropic Fibre and Matrix Assumption

This investigation starts by simplifying the overall investigation by assuming the fibre and the matrix are both isotropic in nature. It is an assumption that is quite commonly made in literature due to its conveniences. This assumption simplifies the number of properties listed in Table 1, from thirteen material properties down to five values (shown in Table 3). The properties for EP280 Prepreg which is the material considered in this investigation are listed in Table 4. Using the micromechanical enhancement process described in the methodology; the micro-mechanical strains within the critical matrix locations were identified, the results are summarised in Table 5.

| Specimen Nº: | $\varepsilon_{11}$ (μ-strain) | $\varepsilon_{22}$ (μ-strain) | $\varepsilon_{33}$ (μ-strain) | $\gamma_{12}$ (μ-strain) | $\gamma_{13}$ (μ-strain) | $\gamma_{23}$ (μ-strain) | $\varepsilon_{dil}$ (μ-strain) |
|---|---|---|---|---|---|---|---|
| 1 | 5405 | -4055 | 4043 | 0 | 8115 | 0 | 28985 |
| 2 | 5551 | -4164 | 4152 | 0 | 8335 | 0 | 29749 |
| 3 | 4667 | -3501 | 3490 | 0 | 7007 | 0 | 25134 |
| 4 | 4593 | -3445 | 3435 | 0 | 6896 | 0 | 24750 |
| 5 | 5358 | -4019 | 4008 | 0 | 8045 | 0 | 28741 |
| Average | 5115 | -3837 | 3826 | 0 | 7680 | 0 | **27472** |

Table 5: Microscopic failure strains obtained for an isotropic fibre and matrix.

### 3.2 Anisotropic Fibre and Isotropic Matrix Assumption

It is known within literature that assuming the fibre to have isotropic properties is often unrealistic [12-14]. However, characterising every one of these material properties is an almost impossible task and there are well presented rules within literature which simplify a lot of them [15, 16]. The inverse rule of mixtures can often be implemented to allow the thirteen material properties for each constituent to be simplified down to eight for the fibre and five for the matrix as given by Table 6. The properties for the fibre and the matrix are shown in Table 7.

| Property | Required for: |
|---|---|
| Volume Fraction | F,M |
| $E_1$ (GPa) | F,M |
| $E_2 = E_3$(GPa) | F |
| $G_{12} = G_{13}$ (GPa) | F,M |
| $G_{23}$ (GPa) | F |
| $\nu_{12} = \nu_{13}$ | F,M |
| $\nu_{23}$ | F |
| $\alpha_1$ (/$^o$C) | F,M |
| $\alpha_2 = \alpha_3$ (/$^o$C) | F |

Table 6: Minimal number of material properties required for the Fibre (F) and Matrix (M).

| Property | Fibre | Matrix |
|---|---|---|
| Volume Fraction | 0.5 | 0.5 |
| $E_1$ (GPa) | 259 | 3.14 |
| $E_2 = E_3$(GPa) | 14 | |
| $G_{12} = G_{13}$ (GPa) | 20 | 1.21 |
| $G_{23}$ (GPa) | 4.83 | |
| $\nu_{12} = \nu_{13}$ | 0.3 | 0.3 |
| $\nu_{23}$ | 0.45 | |
| $\alpha_1$ (/$^o$C) | 0 | $2.38\times10^{-5}$ |
| $\alpha_2 = \alpha_3$ (/$^o$C) | $1\times10^{-5}$ | |

Table 7: Isotropic EP280 neat resin and anisotropic Grafil-34-700 material properties.

The biaxial tensile test results presented in Table 2 were post processed using the new MMEs obtained from running the RVEs with the material properties listed in Table 7. The results are presented in Table 8.

| Specimen N$^o$: | $\varepsilon_{11}$ (μ-strain) | $\varepsilon_{22}$ (μ-strain) | $\varepsilon_{33}$ (μ-strain) | $\gamma_{12}$ (μ-strain) | $\gamma_{13}$ (μ-strain) | $\gamma_{23}$ (μ-strain) | $\varepsilon_{dil}$ (μ-strain) |
|---|---|---|---|---|---|---|---|
| 1 | 5405 | -3255 | 42 | 0 | 974 | 0 | 18478 |
| 2 | 5551 | -3343 | 43 | 0 | 1001 | 0 | 18889 |
| 3 | 4667 | -2811 | 36 | 0 | 841 | 0 | 16401 |
| 4 | 4593 | -2766 | 36 | 0 | 828 | 0 | 16194 |
| 5 | 5358 | -3227 | 42 | 0 | 966 | 0 | 18346 |
| Average | 5115 | -3081 | 40 | 0 | 922 | 0 | **17662** |

Table 8: Microscopic failure strains obtained for an anisotropic fibre and isotropic matrix.

### 3.3 Sensitivity of Anisotropic Assumptions

The previous two subsections encompassed both sides of the spectrum when it comes to predicting matrix failure in composites. In this section, an insight into the importance of assessing which material properties should be given priority during material characterisation are presented.

The properties for the fibre and the matrix were changed in a manner that allowed for a difference to be observed with their effect on the dilatational strain invariant obtained using SIFT. Table 9 summarises the properties that were changed and the magnitude of how much. Note that the base condition is the one in which the fibre was anisotropic and the matrix was assumed to be isotropic. The results were scaled to demonstrate a 10% change in the corresponding properties on the dilatational strain invariant.

| ↑ 10% | Max change in $\varepsilon_{dil}$ |
|---:|:---:|
| Curing temp - stress free temp | 1.15% |
| Thermal expansion coefficient for the fibre | 1.70% |
| Transverse Young's modulus of the fibre | 0.02% |
| Fibre shear modulus | 0.24% |
| Fibre Poisson's ratio (12) | 1.05% |
| Fibre Poisson's ratio (23) | 0.23% |
| Thermal expansion coefficient for the matrix | 8.57% |
| Matrix Young's modulus | 22.44% |
| Matrix Poisson's ratio | 7.01% |
| Fibre volume fraction | 17.82% |

Table 9: Changes in material properties to investigate their effects on SIFT.

## 4 DISCUSSION

In this section, the results presented in Section 3 were refined to examine key material parameters which demonstrated a significant change in the base condition when the fibre was considered anisotropic. Table 9 demonstrated that almost all the matrix properties played a critical role in affecting the dilatational strain invariant obtained using SIFT. These properties include the matrix: thermal expansion coefficient, the Young's Modulus, the Poisson's Ratio and the volume fraction. The Young's Modulus and Fibre Volume Fraction for the composite material and their constituents are known to be critical parameters in any failure prediction model. Thus, their effect on εdil comes as no surprise. However, second to these is the matrix thermal expansion coefficient which was found to change the prediction of εdil given by SIFT by approximately 9% for a 10% change in its property (assuming a linear relationship).

Table 9, also shows that the dilatational strain invariant prediction changes by roughly 7% for a 10% change in the Poisson's Ratio for the matrix. However, it should be noted that this effect is in fact more significant than it appears. This is because, when the Poisson's ratio for the matrix reaches 0.5, indicating that it is an incompressible material (like rubber) its effect on the dilatational invariant is far more significant than the numbers shown in Table 9. An example was performed on the material where the Poisson's ratio for EP 280 was changed from 0.30 to 0.35 and is summarised in Table 10.

| Specimen Nº: | $\varepsilon_{11}$ (µ-strain) | $\varepsilon_{22}$ (µ-strain) | $\varepsilon_{33}$ (µ-strain) | $\gamma_{12}$ (µ-strain) | $\gamma_{13}$ (µ-strain) | $\gamma_{23}$ (µ-strain) | $\varepsilon_{dil}$ (µ-strain) |
|---|---|---|---|---|---|---|---|
| 1 | 6533 | -4563 | 5342 | -2146 | 4068 | -1904 | 16335 |
| 2 | 6679 | -4668 | 5484 | -420 | 4215 | -373 | 16742 |
| 3 | 5795 | -4033 | 4628 | -3881 | 3325 | -3442 | 14284 |
| 4 | 5721 | -3980 | 4558 | 171 | 3252 | 151 | 14079 |
| 5 | 6486 | -4529 | 5297 | -994 | 4021 | -882 | 16205 |
| Average | 6243 | -4355 | 5062 | -1454 | 3776 | -1290 | **15529** |

Table 10: Changes in the dilatational invariant predicted by SIFT with changes in the matrix Poisson's Ratio.

Another property that is an important parameter to consider regarding the anisotropy of the fibres is the transverse Young's modulus. From literature, it is often found that the fibres transverse Young's modulus is significantly smaller than its longitudinal direction [12-14]. This implies that the effect of the transverse Young's Modulus shown in Table 9 becomes significant when the fibre's transverse modulus is changed from 14 GPa to 259 GPa given in Table 7.

All three of these parameters were looked at in more detail with an appropriate analysis performed so that their effects could be observed clearly. Figure 2. Shows a box and whisker plot of the effect of the three properties on the prediction of the dilatational strain invariant. It can be seen that assuming the fibre to be isotropic accounts for the largest change in the critical failure invariant with a change of 55% in the average dilatational invariant.

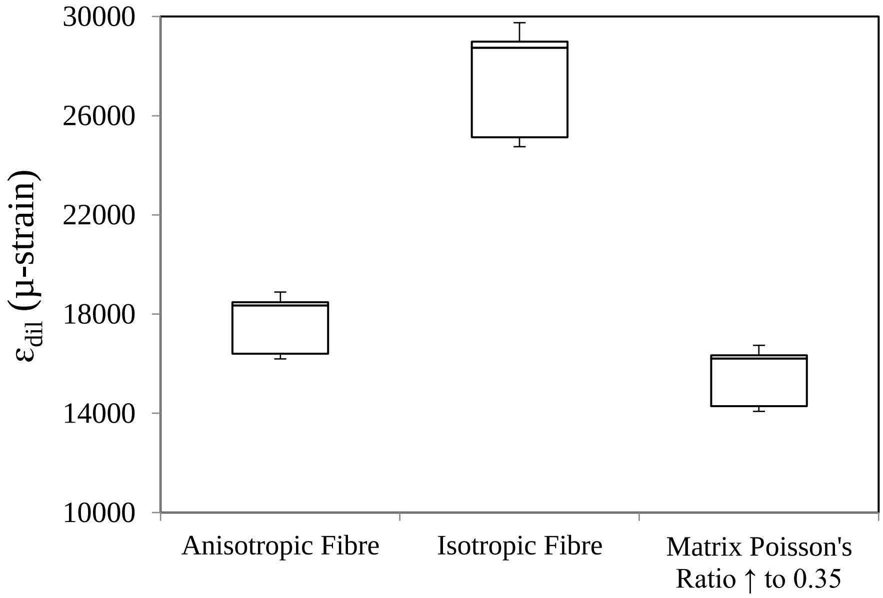


Figure 2: Sensitivity of the dilatational invariant using SIFT to composite material properties.

## 5 CONCLUSIONS

It was found that RVE model predictions for unidirectional fibre composites are particularly sensitive to a small set of material properties, some of which are challenging, if not impossible, to measure reliably. The clear recommendation for this work is that care must be taken when assigning properties to fibre and resin domains in an RVE; the error in material property estimation can swamp any predictive capability. In particular:

- Fibres must be considered as orthotropic materials. Assuming fibre isotropy introduces a 55% error to the RVE predictions for the cases considered here.
- Fibre volume fraction should be measured as accurately as practically possible.
- Thermal expansion is a critical component of accurate RVE modelling. It is likely, although not directly tested here, that related phenomena such as cure shrinkage and moisture expansion are also important modelling considerations.

Overall, the resin properties have a larger influence on the model predictions than the fibre properties; changing the Poisson's Ratio from 0.3 to 0.35 was responsible for a change in the dilatational invariant by 12%. The resin properties should be independently characterised where possible. This can pose a challenge for those using proprietary prepreg composite systems as neat resin is often challenging to acquire.

Fibre properties (in particular transverse properties) are challenging, if not completely impractical, to characterise for most applications. In most cases these properties are obtained by "rule-of-mixtures" approaches which interpolate fibre properties from resin and laminate properties. A recommendation, based on this work, is that the most critical parameter to characterise for a fibre is the transverse thermal expansion coefficient.

## REFERENCES


[1] M.-h. Yu, "Advances in strength theories for materials under complex stress state in the 20th Century," *Applied Mechanics Reviews*, vol. 55, pp. 169-218, 2002.

[2] J. H. Gosse and S. Christensen, "Strain invariant failure criteria for polymers in composite materials," 2001, pp. 45-55.

[3] R. Pipes and J. Gosse, "An onset theory for irreversible deformation in composite materials," in *ICCM-17, the 17th international conference on composite materials, Edinburgh, UK*, 2009, p. 31.

[4] G. P. S. H. Lim, D. Kelly, B. G. Prusty, A. Crosky, "NEW DEVELOPMENTS IN ONSET THEORY FOR ONSET OF RESIN FAILURE IN FIBRE REINFORCED COMPOSITES," *19th International Conference on Composite Materials*, pp. 5551-5558, 2012.

[5] N. T. Chowdhury, "A novel failure criterion for matrix dominant failure within fibre reinforced polymer composites," Doctor of Philosophy, Mechanical and Aerospace Engineering, Monash University. Faculty of Engineering. Department of Mechanical and Aerospace Engineering, 2016.

[6] N. T. Chowdhury, W. K. Chiu, and J. Wang, "Biaxial Specimen Design for Structures Subjected to Multi-axial Loads," *Advanced Materials Research*, vol. 891-892, pp. 1633-1638, 2014.

[7] N. T. Chowdhury, W. K. Chiu, J. Wang, and W. Yan, "Comparison of failure criterion examining matrix dominant failure under biaxial loading," presented at the 8th Australasian Congress on Applied Mechanics: ACAM 8, Melbourne, Australia, 2014.

[8] N. T. Chowdhury, J. Wang, and W. K. Chiu, "Design of a Flat Plate Specimen Suitable for Biaxial Tensile Tests on Polymer Materials," *Polymer and Polymer Composites*, vol. 23, pp. 627-638, 2015.

[9] N. T. Chowdhury, J. Wang, W. K. Chiu, and W. Yan, "Matrix failure in composite laminates under tensile loading," *Composite Structures,* vol. 135, pp. 61-73, 1// 2016.

[10] T. M. Tran and F. o. E. U. Manufacturing Engineering, "Development of micromechanical modelling procedures using the onset theory for failure of composites," D. M. Kelly, F. o. E. U. Manufacturing Engineering, G. M. Prusty, and F. o. E. U. Manufacturing Engineering, Eds., ed, 2012.

[11] L. J. Hart-Smith, "Application of the strain invariant failure theory (SIFT) to metals and fiber–polymer composites," *Philosophical Magazine*, vol. 90, pp. 4263-4331, 2010/11/07 2010.

[12] T. Z. N. Sokkar, M. A. Shams El-Din, and A. S. El-Tawargy, "On Young's modulus profile across anisotropic nonhomogeneous polymeric fibre using automatic transverse interferometric method," *Optics and Lasers in Engineering,* vol. 50, pp. 1223-1229, 9// 2012.

[13] M. Riley and J. Whitney, "Elastic properties of fiber reinforced composite materials," *Aiaa Journal*, vol. 4, pp. 1537-1542, 1966.

[14] S. Kawabata, "Measurement of the transverse mechanical properties of high-performance fibres," *Journal of the Textile Institute,* vol. 81, pp. 432-447, 1990.

[15] I. M. Daniel and O. Ishai, *Engineering Mechanics of Composite Materials*: Oxford University Press, 2006.

[16] C. Sun and R. Vaidya, "Prediction of composite properties from a representative volume element," *Composites Science and Technology,* vol. 56, pp. 171-179, 1996.