DAMAGE DETECTION FOR COMPOSITES USING A BAYESIAN REGULARIZATION-BASED ELECTRICAL IMPEDANCE TOMOGRAPHY TECHNIQUE

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ABSTRACT

Electrical impedance tomography (EIT) is a non-radiative and low-cost imaging technique that aims to estimate the interior electrical properties of an object from current-voltage measurements on its boundary. Recently, with the rapid development of nanotechnology, EIT has been applied to damage detection for composites combined with the excellent electrical properties of carbon nanotubes (CNTs). In this work, an adaptive Bayesian regularization algorithm is proposed for EIT to identify quantitatively the damage in composites. By hierarchical Bayesian modeling, the *posterior* probability distribution of conductivity change caused by damage is established within the context of Bayesian inference. Through a Bayesian learning algorithm, a *maximum a posteriori* (MAP) solution is adopted to automatically determine the optimal value of the regularization parameter, and reconstruct the unknown distribution of conductivity change, which quantitatively indicates the damage location and size. Numerical studies for synthetic data from a finite element (FE) model and experimental studies for a glass fiber reinforced polymer (GFRP) plate with a CNT sensing skin have been performed to demonstrate the applicability and effectiveness of the proposed Bayesian regularization-based EIT approach.

1 INTRODUCTION

Composites have been widely used in structural components of aerospace vehicles. However, they are vulnerable to damages, especially the invisible low-impact damage, leading to significant reductions of strength and reliability. Therefore, it is important to develop structural health monitoring (SHM) technologies for composites to detect the internal damages at an early stage to prevent catastrophic failure [1]. In recent years, the rapid development of nanotechnology has provided an attractive way for SHM of composites. The excellent electrical properties of carbon nanotubes (CNTs) enable composites equipped with CNTs as nano fillers or with CNT films as sensing skins to have the ability of "self-sensing", which can be utilized for detecting damage through conductivity/ resistivity changes without any additionally installed sensors [2,3].

Among those conductivity/resistivity-based methods, electrical impedance tomography (EIT) which originated from medical imaging, is a promising technique for its advantage of low cost, non-radiation, high speed, and capability of quantitatively identifying the damage [4-6]. Nonetheless, EIT is typically a well-known ill-posed inverse problem which is vulnerable to measurement noise and whose solution may be unstable. Thus, regularization is usually required to solve the inverse problem with penalty terms so as to obtain a stable bounded solution and reduce artefacts. The most widely used mathematical regularization in EIT is the Tikhonov regularization which relaxes the ill-condition by balancing the original objective function and a smoothness condition on the solution [7]. The major difficulty of applying Tikhonov's regularization lies in finding the optimal regularization parameter efficiently. A number of choices are available in the literature, e.g., the *L*-curve method [8] and the generalized cross-validation (GCV) method [9]. These methods operate on the basis of the singular value decomposition (SVD) of the system matrix and are only computationally effective when the scale/size of the formulated linear equations is not very large. Nevertheless, when the matrix size increases, the computational efficiency decreases dramatically. Large values of the matrix size bring severe computation burden when using the *L*-curve and GCV methods, especially in an iterative optimization process where repeated forward analyses are required. To overcome the deficiencies of the traditional Tikhonov regularization, Jin and Zou [10] developed a regularization approach based on Bayesian inference which could adaptively determine the regularization parameter and detect the noise level in a data-driven manner. Bayesian regularization has been successfully used for identification of traffic excitations for truss structures and impact forces for composite structures, demonstrating its potential in solving ill-posed inverse problems in SHM [11,12].

In this study, the adaptive Bayesian regularization algorithm is combined with EIT to quantitatively identify the damage in composites. By hierarchical Bayesian modeling, the *posterior* probability distribution of conductivity change caused by damage is established within the context of Bayesian inference. A *maximum a posteriori* (MAP) solution is adopted to automatically determine the optimal value of the regularization parameter, and to reconstruct the unknown distribution of conductivity change, which is used to indicate quantitatively the damage location and size. Both numerical and experimental studies are performed to validate the proposed Bayesian regularization-based EIT approach.

2 ELECTRICAL IMPEDANCE TOMOGRAPHY FOR DAMAGE DETECTION

EIT is a soft-field tomographic imaging method that uses boundary voltage measurements from flowing electric current to reconstruct the internal electrical property (usually conductivity or resistivity) distribution of an object. Figure 1 is an illustration of EIT. To generate a tomographic image by using EIT, two problems, i.e., the forward problem and inverse problem of EIT should be solved.

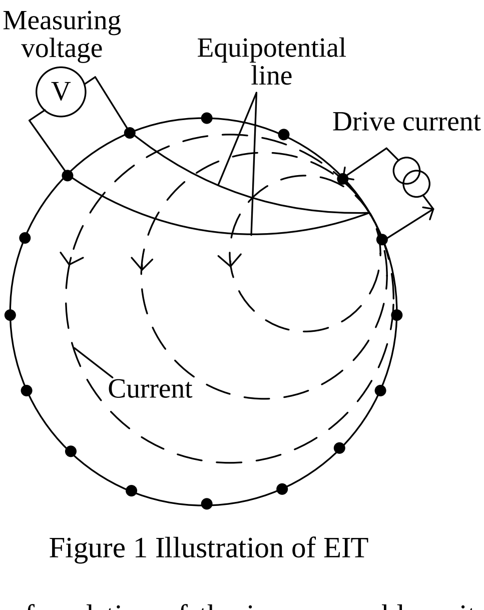


Figure 1 Illustration of EIT

The forward problem is the foundation of the inverse problem, it solves for a domain electrical potential u using a known conductivity distribution σ. The governing equation of the forward problem is a Laplacian equation derived from Maxwell's equations as:

$$\nabla \cdot (\sigma \cdot u) = 0 \tag{1}$$

In equation (1), it is assumed that no current source exists in the interior of the domain.

Usually, for an object of arbitrary shape, analytical solution cannot be obtained for the forward problem, thus finite element method (FEM) is used to solve equation (1). To consider the boundary conditions, the complete electrode model (CEM) which defines the total amount of current and voltage at boundary electrodes and for the rest of the domain is applied [13]. After discretization, the FEM equations of CEM can be written as:

$$\begin{bmatrix} A_m + A_z & A_w \\ A_w & A_D \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{2}$$

where U is a vector of domain potentials, V is a vector of potentials on the electrodes and I is a vector of current injections, A_m is the global diffusion stiffness matrix assembled by local diffusion stiffness matrices of each element, A_z, A_w and A_D, imposed on the CEM boundary conditions.

The inverse problem of EIT aims to obtain the conductivity distribution supplied to the forward operator until the difference between the experimentally measured voltages and the computationally predicted voltages is minimized in the least-squares sense. That is,

$$\sigma^* = \min_{\sigma} \left(\left\| V_m - F(\sigma) \right\|^2 \right) \tag{3}$$

where V_m is the vector of experimentally measured voltages, and F the forward operator which is usually performed by FEM. When the background conductivity distribution σ_0 is known, $F(\sigma)$ can be approximated by a Taylor series expansion centered about σ_0, and then equation (3) which is a nonlinear inverse problem can be transformed to a linear inverse problem to obtain the conductivity change as:

$$\Delta\sigma^* = \min_{\Delta\sigma} \left(\left\| \Delta V - J \Delta\sigma \right\|^2 \right) \tag{4}$$

where $\Delta V = V_m - F(\sigma_0)$, $\Delta\sigma = \sigma - \sigma_0$, and $J = \dfrac{\partial F(\sigma_0)}{\partial \sigma}$ is known as the sensitivity matrix which can be formed by Geselowitz's theorem for each element as [13]:

$$J_{ij} = \int_{\Omega_e} \nabla u_i \cdot \nabla u_j d\Omega_e \tag{5}$$

where u_i is the voltage for the i^{th} injection electrode pair with unit current and u_j is the voltage when the j^{th} measurement electrode pair is carrying a unit current.

3 RECONSTRUCTION OF CONDUCTIVE CHANGE BY BAYESIAN REGULARZIATION

In general, it is not straightforward to reconstruct the conductivity change by a direct matrix inversion from equation (4), since the least-squares problem may be ill-posed (e.g., J is singular) when measurement noise and modeling error are present. To provide a bounded solution, a regularization term should be added to equation (4) while the ill-posed problem is solved as:

$$\Delta\sigma^* = \arg\min_{\Delta\sigma} \left(\left\| \Delta V - J \Delta\sigma \right\|^2 + \lambda \left\| L \Delta\sigma \right\|^2 \right) \tag{6}$$

where L is the regularization matrix, and λ a non-negative regularization parameter. When the unit matrix I is used as the regularization matrix, it is the well-known Tikhonov regularization, and the solution can be obtained by:

$$\Delta\sigma^* = \left(J^T J + \lambda I \right)^{-1} J^T \Delta U \tag{7}$$

Nonetheless, as mentioned in the Introduction, the difficulty of applying the Tikhonov regularization lies in how to efficiently find the optimal regularization parameter λ. In this study, a statistical analysis within the context of Bayesian inference is employed to automatically determine an optimal value of the regularization parameter λ, and reconstruct the unknown conductivity change iteratively through a statistical Bayesian learning scheme.

By hierarchical Bayesian modeling, the *posterior* probability distribution of conductivity change $\Delta\sigma$ due to damage can be written as:

$$p\left(\Delta\sigma, \tau^2, \eta^2 \mid \Delta V\right) \propto p\left(\Delta V \mid \Delta\sigma, \tau^2\right) p\left(\Delta\sigma \mid \eta^2\right) p\left(\tau^2\right) p\left(\eta^2\right) \tag{8}$$

in which the likelihood function,

$$p\left(\Delta V \mid \Delta\sigma, \tau^2\right) \propto \frac{1}{\tau^{N_m}} \exp\left(-\frac{1}{2\tau^2}\left\|J\,\Delta\sigma - \Delta V\right\|^2\right) \tag{9}$$

describes the probability of observing the measured change of boundary voltages ΔV , and the prior

$$p\left(\Delta\sigma \mid \eta^2\right) \propto \frac{1}{\eta^{N_e}} \exp\left(-\frac{1}{2\eta^2}\left\|\Delta\sigma\right\|^2\right) \tag{10}$$

shows prior knowledge about $\Delta\sigma$. The remaining two terms in equation (8) are the Gamma priors of the hyper-parameters τ^2 and η^2 which are:

$$p\left(\tau^2\right) \propto \frac{\beta_0^{\alpha_0}}{\Gamma\left(\alpha_0\right)} \tau^{-2\left(\alpha_0+1\right)} e^{-\beta_0\tau^{-2}} \tag{11}$$

$$p\left(\eta^2\right) \propto \frac{\beta_1^{\alpha_1}}{\Gamma(\alpha_1)} \eta^{-2\left(\alpha_1+1\right)} e^{-\beta_1\eta^{-2}} \tag{12}$$

Substituting equations (9)-(12) into equation (8), an analytical expression as follows can be obtained:

$$p\left(\Delta\sigma, \tau^2, \eta^2 \mid \Delta V\right) \propto \frac{\eta^{-2(\alpha_1+1)-N_e}}{\tau^{2(\alpha_0+1)-N_m}} \exp\left(-\frac{1}{2\tau^2}\left\|J\,\Delta\sigma - \Delta V\right\|^2 - \frac{1}{2\eta^2}\left\|\Delta\sigma\right\|^2 - \beta_0\tau^{-2} - \beta_1\eta^{-2}\right) \tag{13}$$

Here, a MAP solution is considered by maximizing $p\left(\Delta\sigma, \tau^2, \eta^2 \mid \Delta V\right)$, which is:

$$\left\{\Delta\sigma, \tau^2, \lambda^2\right\} = \arg \max_{\left\{\Delta\sigma, \tau^2, \lambda^2\right\}} \left\{p\left(\Delta\sigma, \tau^2, \lambda^2 \mid \Delta V\right)\right\}. \tag{14}$$

It is equivalent to solving a set of three optimization equations (15)-(17) below:

$$\left(J^T J + \frac{\tau^2}{\eta^2}\right)\Delta\sigma - J^T \Delta V = 0 \tag{15}$$

$$\left[N_m + 2\left(a_0 + 1\right)\right]\tau^2 - \left\|J^T \Delta\sigma - \Delta V\right\|^2 - 2\beta_0 = 0 \tag{16}$$

$$\left\|\Delta\sigma\right\|^2 + 2\beta_1 - \left[N_e + 2\left(\alpha_1 + 1\right)\right]\eta^2 = 0 \tag{17}$$

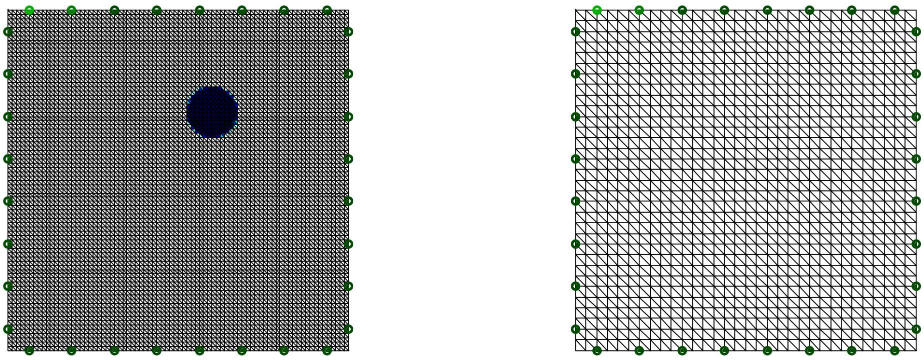
After equations (8)-(10) are solved, the conductivity change $\Delta\sigma$ due to damage can be reconstructed and an optimal value of the regularization parameter can be obtained as:

$$\lambda = \tau^2 \big/ \eta^2 . \tag{18}$$

4 NUMERICAL STUDY

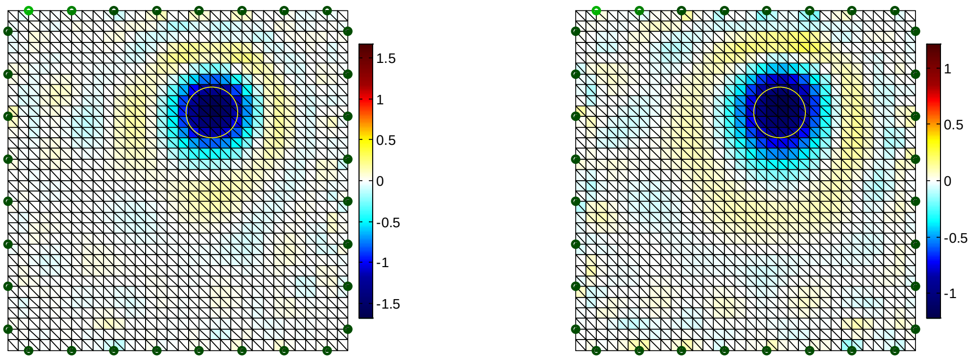
To verify the adaptive Bayesian regularization-based EIT method, a series of numerical studies are performed. The synthetic data are generated by an open software EIDORS [14] through MATLAB (R2009a, MathWorks®). Figure 2(a) illustrates a forward FE model of a square domain with normalized unit dimensions for generating the synthetic data. It contains 12800 elements and 32 electrodes which result in 928 boundary voltage measurements when adjacent injection and measurement modes are adopted. The background conductivity is 5 S/m, and a circular damage is presented with an assumed conductivity reduction of 2 S/m. Figure 2(b) shows the inverse FE model for reconstruction of the distribution of conductivity change due to this damage. The mesh of the inverse model is much coarser and contains only 2048 elements. This coarse mesh can help increase the reconstruction efficiency and avoid “inverse crime”. Gaussian white noise with two different signal-to-noise ratio (SNR), i.e., SNR = 30 dB and SNR = 20 dB, is added to the synthetic data to simulate the effects of measurement noise.

The developed Bayesian regularization-based EIT algorithm is applied to the noise-contaminated synthetic data, and results are obtained for the reconstruction of the conductivity change to quantitatively indicate the damage. Figures 3(a) and (b) show the tomographic image of the reconstructed conductivity change for the synthetic data with noise level of SNR = 30 dB and SNR = 20 dB, respectively. The true location and size of the damage are labelled on the images for comparison. It can be seen that the damage is well indicated by the localized area with conductivity reduction. The identified location matches well that of the simulated damage, and the identified size is a little larger than the true size due to the 'soft' nature of the electrical field and the noise effect. Also, from the Bayesian algorithm, the optimal regularization parameter can be adaptively determined through the data without any manual manipulation as illustrated in Figures 4(a) and (b), demonstrating the effectiveness of the proposed adaptive Bayesian regularization-based EIT method.



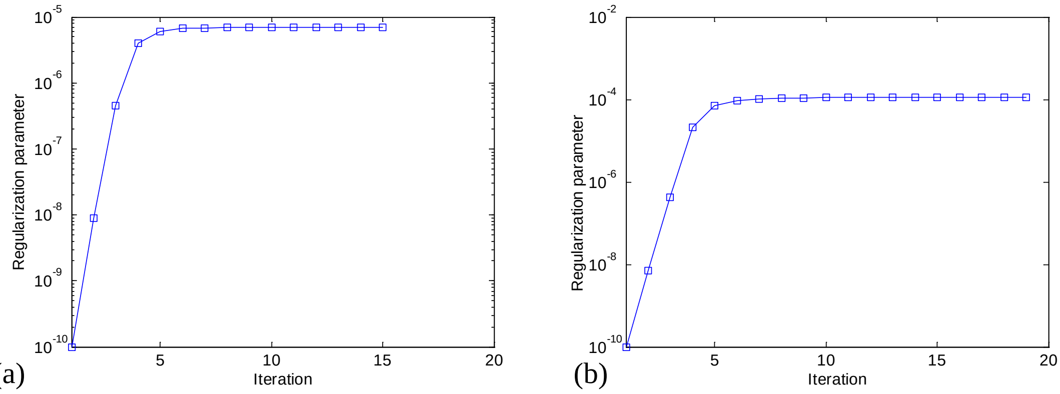
(a) (b)

Figure 2 Finite element models in the numerical studies: (a) forward model, and (b) inverse model



(a) (b)

Figure 3 Reconstructed conductivity change for the numerical cases with noise level of (a) SNR = 30 dB and (b) SNR = 20 dB



(a) (b)

Figure 4 Regularization parameter for the numerical cases with noise level of (a) SNR = 30 dB and (b) SNR = 20 dB

5 EXPERIMENTAL SUTDY

In the experimental study, a 2 mm-thick glass fiber reinforced polymer (GFRP) plate with a CNT film (JCNTF-20C, JCNANO Tech Co., Ltd) as shown in Figure 5(a) is considered. The dimensions of the CNT film are 10 ×10 cm^2, and it is attached on the surface of the GFRP plate by epoxy. 20 electrodes coated with a conductive silver paint are equally spaced along the boundary of the film, and they are connected to a data acquisition system. A constant DC current of 100 mA generated by Keithley 6221 high precision current source is applied to the film between the adjacent electrode pair sequentially and corresponding voltage measurements are acquired from the rest of the electrode pairs by Keithley 2701 multi-meter. The test procedure is first performed under the pristine state, i.e., the CNT film and the GFRP plate are undamaged, to obtain a set of reference voltage measurements. Then, the damage is introduced and the same testing procedure is performed to obtain voltage measurements under the damaged state. Figure 5(b) shows the GFRP plate with a circular hole damage whose radius is 12 mm.

EIT is then used to reconstruct conductivity change of the CNT film caused by damage with these boundary voltage measurements. The inverse FE model for reconstruction of the conductivity change is illustrated in Figure 6(a) which contains 3200 elements. Figure 6(b) shows the tomographic image of the conductivity change corresponding to Figure 5(b) reconstructed by a normalized version of the proposed algorithm. Similar to the numerical cases, the damage is well indicated by the localized area with a significant conductivity reduction. In addition, the optimal regularization parameter is adaptively determined through the data illustrated in Figure 7, further demonstrating the effectiveness of the proposed adaptive Bayesian regularization-based EIT method.

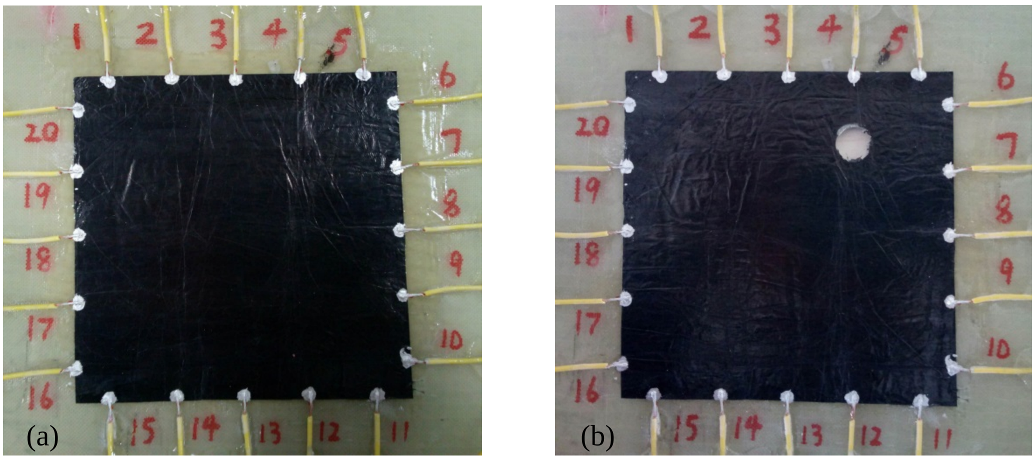
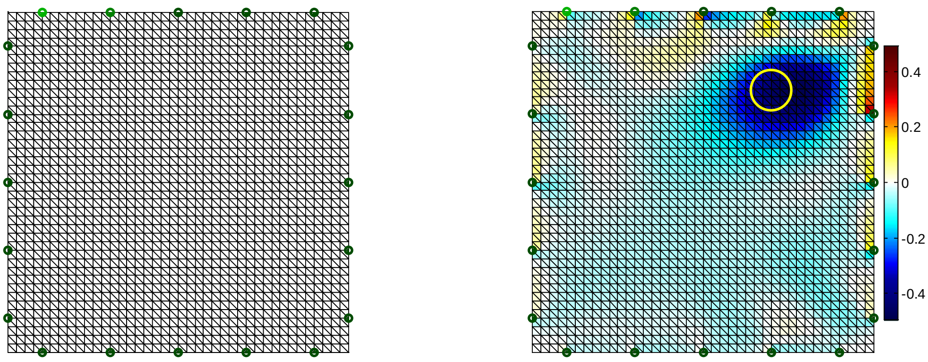


Figure 5 GPRP plate with CNT film as a sensing skin (a) without damage and (b) with damage



(a) (b)

Figure 6 EIT for the experimental case: (a) inverse model and (b) reconstructed conductivity change

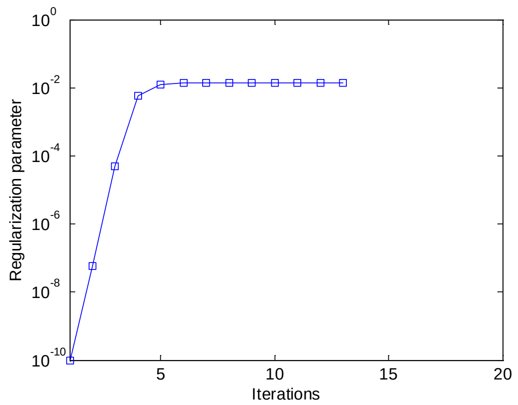


Figure 7 Regularization parameter for the experimental case

6 CONCLUSIONS

In this study, an adaptive Bayesian regularization algorithm is proposed for EIT to quantitatively identify the damage in composites. By hierarchical Bayesian modeling, the *posterior* probability distribution of conductivity change is established within the context of Bayesian inference, and a MAP solution is adopted to reconstruct the unknown distribution of conductivity change. The developed Bayesian regularization-based EIT method is first applied to a numerical example and then to an experimental example of a GFRP plate with CNT film as its sensing skin. Both numerical and experimental results have shown that the proposed approach can adaptively determine the optimal regularization parameter for the ill-posed EIT inverse problem from the measurement data and successfully reconstruct the conductivity change caused by damage, demonstrating its potential in SHM of composite structures.

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