NUMERICAL SIMULATION OF CHARACTERISTIC OF RESIN INFUSION AT MICRO-SCALE

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Key words: Infusion, Fiber, Micro-scale, Numerical simulation

ABSTRACT

Liquid Composite Molding (LCM) is a high-efficiency and low-cost method for composites manufacture. In LCM, the permeability of the porous composite preform is a crucial and intrinsic property. In literatures, the inner structure inside the fiber tows is considered to be decisive to the permeability. In this paper, a random configuration model for fiber arrangement is proposed and implemented in a computer code. This development enables the investigation of the effect of the random arrangements of the fiber on the permeability and the flow properties. Five parameters are considered including four micro-structural parameters (L, r, Δr and δ_{min}) and one macro parameter (ε). Numerical simulation is performed for the transverse flow of media with random arrangements of fibers. Morris method for Global Sensitivity Analysis (GSA) is used to study the influence of these parameters. The results shown that the porosity ε has the most obvious influence on the permeability. Among all parameters, L plays a dominant role on influencing the average velocity.

1 INTRODUCTION

The core step of LCM method is the impregnation process. Thus, it is important to explore the flow resistance during the infusion process. The flow ability of liquid through a porous media is described by its permeability. The permeability is a second order tensor. It influences the infusion degree [1, 2] as dictated by Darcy's law.

The numerical simulations of the resin infusion process have been performed at the macro-scale or at dual-scales. The macro-scale approach considers the flow through the gaps between the tows whereas the dual-scale approach considers both the saturated flow through the gaps between the tows and the unsaturated flow inside the tows.

For example, Brinkman equation [3] is a macro-scale approach. It is a classical method to describe the flow in porous media. However, the fiber tows are considered as whole, and the complex flow inside the tows is ignored [4].

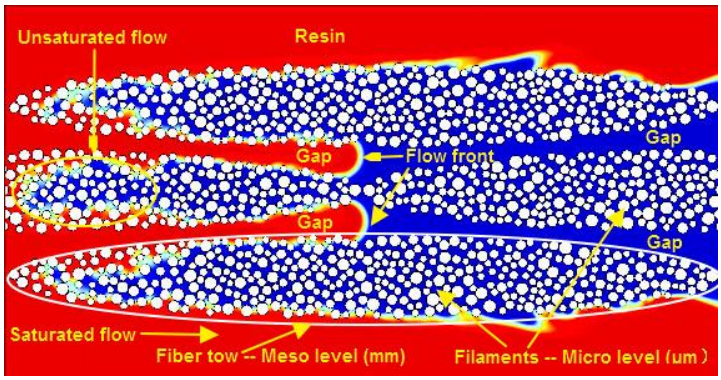


Figure 1: Flow status at the meso-micro level.

The dual-scale models have also been developed. Gebart [5] developed analytical equations for permeability of ideal quadratic and hexagonal arrangement of fibers for both longitude and transverse flows. Lundström and Gebart [6] set up models for the permeability of several types of perturbation arrangement of fibers based on the lubrication approximation theory. The above works were on preforms with regularly arranged structures. M. Q.Thai, F.Schmidt et al.[7] applied BEM (Boundary Element Method) to obtain numerical results on a square packed set of fibers and compare with experiments. For preforms with random arrangement, Chen and Papathanasiou [8] also used BEM to investigate the transverse flow across random arrangement fiber structure and set up a correlation between the mean nearest inter-fiber distance with the permeability. K.Yazdchi [9] proposed a relation for predicting the permeability for random arranged fiber in terms of the micro-structural average channel width.

The literature review indicates that the flow property based on model with constant radii of fibers has been studied. However, the perturbation arrangement of the fibers and the influence of micro structural parameters have not been explored.

The objective of this study is to investigate the interaction of the micro structural parameters and macro parameter on the permeability and to compare the influence of these parameters.

2 GEOMETRICAL MODEL SET UP

2.1 Parameters of the geometrical model

REV (Representative Elementary Volume) method is often used to explore the flow inside the tows. RVE are often built for a regular distribution of fibers and sizes of fibers. In a real manufacturing process, however, perturbations of fiber arrangement are unavoidable. Since the micro-architecture of the fiber arrangement will affect the micro flow inside the tow, it will have a substantial influence on the infusion process.

The geometrical model used in this work have five parameters, as shown in figure (Fig. 2) below. δ represents the distance between each two fibers. δ_{min} is defined as the minimum distance between two fibers among all the fibers. $r \pm \Delta r$ is the range of the fiber radius. r is the radii of the fibers, normally, the average radius of one filament inside the tow is 3.5 μm. Here r is considered as a constant with the value of 3.5 μm.

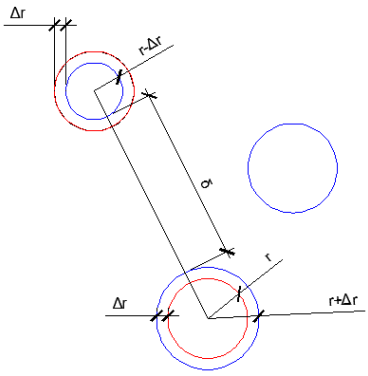


Figure 2: Definition sketch of an arrangement of fibers with perturbation of fiber radii.

The flow status can be determined by porosity which is a crucial macroscopic parameter. In the 2D domain, porosity can be calculated as:

$$\varepsilon = 1 - \frac{\pi \sum_{i=1}^{n} r_n^2}{L^2} \tag{1}$$

where ε is the porosity; r_n is the radii of the fiber filaments inside a tow, which takes into account variations of the sizes of realistic fiber filaments; and L is the length of the square domain.

2.2 Mechanism of random radii and random positions

A computer program based on the method of a refined random point in region is used to produce random positions and random radius of the fiber. A schematic of the code is shown below in figure 3.

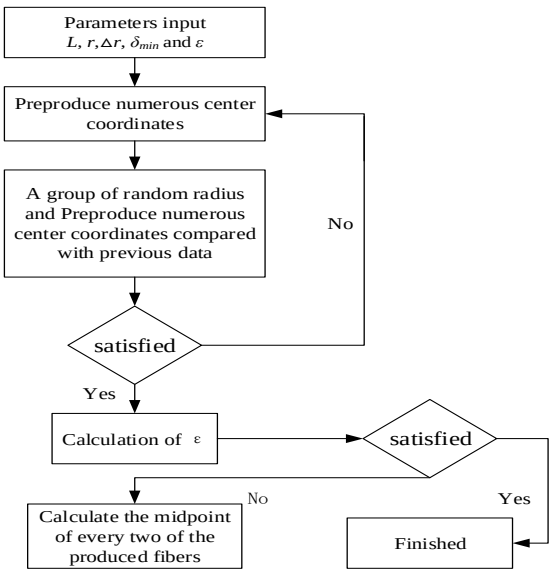
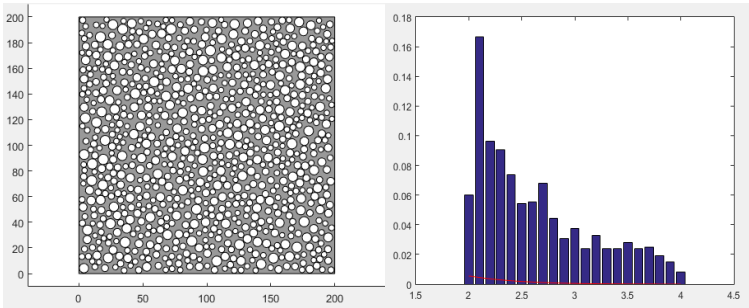


Figure 3: Schematic of the random fiber code.

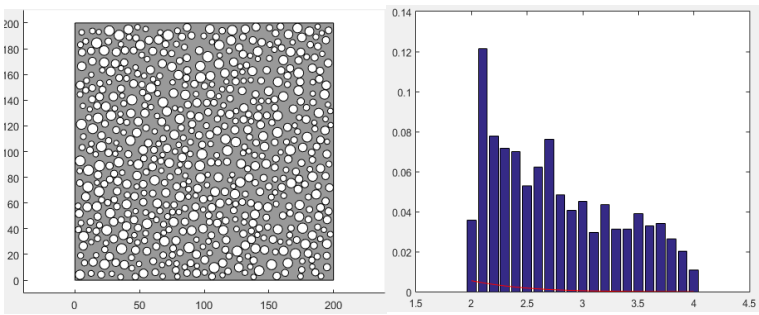
L, r, Δr, δ_{min} and ε are five input parameters. At first, numerous center coordinates in a $L*L$ domain are randomly produced and stored. With the input of Δr, the range of the radii is determined. Each time when a group of data including center coordinates are taken out from the preproduced structure, random radius among the range of $r \pm \Delta r$ are produced. Each new group of random fiber data will be estimated by previous decided fiber according to δ_{min}. If this group of data satisfies the requirement, it will be

confirmed. Otherwise the computation will carry on the loop to next one. At the same time, ε is computed when a new fiber is decided, the code will iterate until the computed value of ε meets the requirement. To improve the efficiency of the calculation, the calculated midpoint of every two of the produced fibers is judged to determine whether this midpoint can be the new center of the fiber which can make the distribution of the fibers more compact.

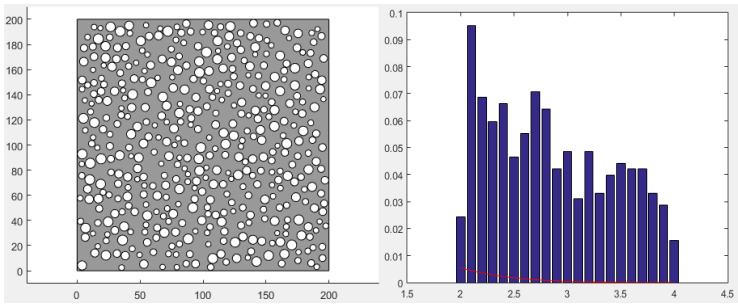
The radii and the distribution of the fibers are generated randomly by the program by controlling the length of the domain, the minimum inter fiber distance and the porosity. Since the porosity and the domain is given, the total area of the fiber is limited, especially when the porosity is as low as 0.5, the "space" of the domain is controlled and the fibers are much more concentrated on small radii. The distribution of the radii of fibers in high porosity model are more homogeneous. There is more "space" available of equal probability for large and small radii.



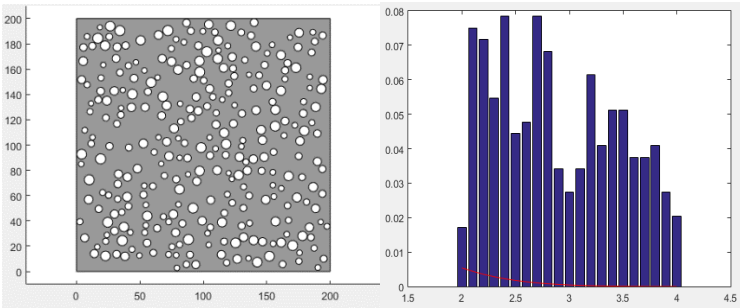
(a) $\varepsilon = 0.5$ (Fiber number: 891)



(b) $\varepsilon = 0.6$ (Fiber number: 643)



(c) $\varepsilon = 0.7$ (Fiber number: 437)



(d) ε = 0.8 (Fiber number: 293)

Figure 4: Arrangement of the fiber in the 2D square domain and distribution of the sizes of the radii

2.3 The determination of the value range of the parameters

As discussed above, L, r, Δr, δ_{min} and ε are the five factors defined in the model. In our model, ε is the only macro parameter, and it has direct interaction with δ_{min}. ε will decrease with the increase of micro parameter δ_{min}, since all these five parameters are defined in the input. According to the limitation of ε and L, δ_{min} cannot be too large. In addition, when L is 500 μm and ε is 0.53, the upper limit of the total number fibers is 6323. Though this is needed for our simulation, the calculation cost is relatively high. The minimum porosity ε for random placement method in 2D case has found to be 0.453 according to [10], and thus the range of ε is set at 0.5 to 0.95 in our study. Since r=3.5 μm, Δr in the range of 0.5 μm to 3 μm is appropriate. By practice of test, the ranges of these four parameters are set as below in table 1.

r	[μm]	3.5
Δr	[μm]	0.5-3
δ_{min}	[μm]	0.1-1
L	[μm]	100-500
ε		0.5-0.95

Table 1: Range of the parameters of the model

3. MATHEMATICAL FORMULATION AND NUMERICAL METHODOLOGY

3.1 Output results and analysis.

Under creeping flow conditions, the classical Darcy's law can be expressed in the form below.

$$\boldsymbol{u} = -\frac{\boldsymbol{K}}{\mu}\nabla P \tag{2}$$

where $\boldsymbol{u}$ is the superficial velocity, and it is related to linear velocity U as $\boldsymbol{u}=\varepsilon\cdot U$, $\boldsymbol{K}$ is the permeability tensor of the fiber intra tow, μ is the viscosity of the fluid and P is the pressure. In a unit domain, if the pressure difference is Δp and the length of the flow is L, the Darcy's velocity can be expressed as:

$$\boldsymbol{u} = \frac{\boldsymbol{K}}{\mu}\cdot\frac{\Delta p}{L} = \varepsilon\cdot U \tag{3}$$

Where U is calculated by $U = \frac{\int_{\Gamma_S}\vec{v}\cdot\vec{n}d\Gamma}{H}$, and length of the flow L is equal to the outlet length H in a square domain, then the transverse permeability is calculated as shown:

$$K = \frac{\mu\varepsilon}{P_{in}-P_{out}} \int_{\Gamma_s} \vec{v} \cdot \vec{n} d\Gamma \tag{4}$$

where P_{in} and P_{out} are the pressures of the inlet and outlet, Γ_s is the outlet. $\vec{v}$ is the velocity of the resin and $\vec{n}$ is the unit normal vector.

Meanwhile, since the Reynolds number of the resin infusion process is sufficiently low to satisfy the laminar flow condition (or exactly the creeping flow), and the fluid is supposed to be incompressible, the average velocity of the outlet is calculated by the simplified Navier-Stokes equation (eq. 5) and continuity equation (eq. 6) can be used to carry out the simulation [11]:

$$\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla \cdot [-p\boldsymbol{I} + \mu(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)] + \boldsymbol{F} \tag{5}$$

$$\rho\nabla \cdot (\boldsymbol{u}) = 0 \tag{6}$$

where ρ, $\boldsymbol{u}$, p, μ, $\boldsymbol{F}$, $\boldsymbol{I}$ are density, velocity, pressure, volume force and unit matrix respectively.

3.2 Boundary conditions and initial values

For a square domain, there is an inlet boundary and an outlet one. Symmetry conditions are adopted on the other two boundaries. No slip condition is set on the fiber contours. Boundary conditions and intial values are show as below in figure 5 and table 2.

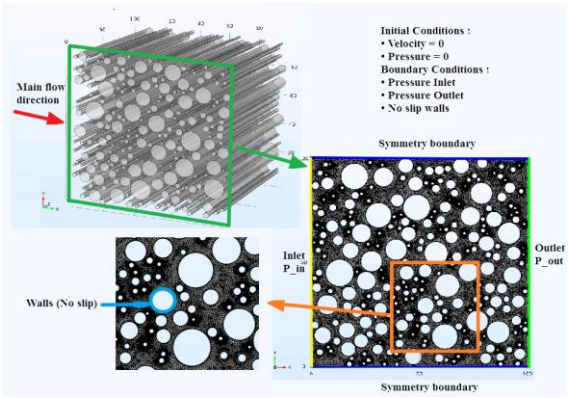


Figure 5: Boundary conditions of the model

P_{in} [Pa]	P_{out} [Pa]	μ [Pa.s]	Initial velocity V_x [m/s]	Initial velocity V_y [m/s]	Initial Pressure P_0 [Pa]
100000	0	0.1	0	0	0

Table 2: Values on the boundaries

The velocity field distribution of one of the models obtained by our method is presented below in figure 6. In this model, there are 6323 fibers in a square domain with the side length of 500 μm.

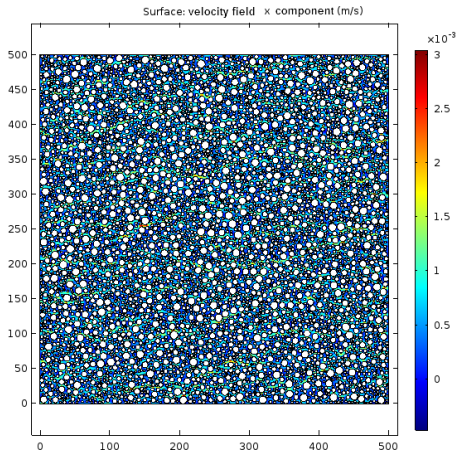


Figure 6: Velocity field of the model
(δ_{min} = 0.1 μm, Δr = 3 μm, L=500 μm, ε =0.53, fiber number: 6323)

3.3 Results compared with earlier research

Since the fiber radii are random in a range and the position is also random in the current study, Kozeny-Carman equation and Gebart equation are not suitable for the comparison. Chen and Papathanasiou [7] developed a similar model with the constant radii of the fiber. They defined 576 fibers in a square domain with the porosity and the minimum inter fiber distance to generate the model. In their numerical experiments, $\boldsymbol{K}/r^2$ is adopted to describe the permeability, since it can be turned to be a dimensionless value. In our case, the radius r is 3.5 μm, to adapt to their form, Δr = 0 μm. The detailed data is listed in table 3. The velocity field distribution is shown in figure 7.

	ε	Δr [μm]	δ_{min} [μm]	r [μm]	K [m^2]
Chen et al.	0.7	0	$0.1r$	3.5	1.15×10^{-12}
Our work	0.7	0	$0.1r$	3.5	8.33×10^{-13}

Table 3: Comparison of the results with X. Chen

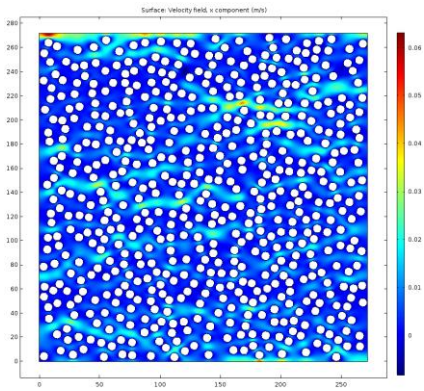


Figure 7: Velocity field distribution of the compared model
(δ_{min} = 0.35 μm, Δr = 0 μm, ε = 0.7, number of the fiber=576)

For comparison, numerical simulation results of Sangani and Yao [12], Chen and Papathanasiou [8] are included as presented in figure 8.

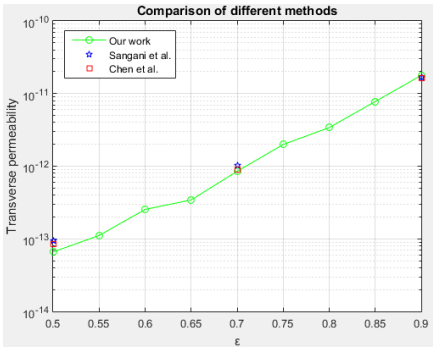


Figure 8: Comparison with different methods

3.4 Influence of each parameter on the permeability

Variation of radii Δr

The relationship between the transverse permeability and Δr is shown in figure 9. With the increase of the range Δr, the permeability shows a declining trend. This can be explained as the following. When the range of Δr is large, the difference between the largest fiber and the smallest one is huge. Under the condition of a low porosity such like 0.5, the ratio of small size fibers is high (Fig. 4a). Thus, the "flow channel" is easier to be blocked by small fibers.

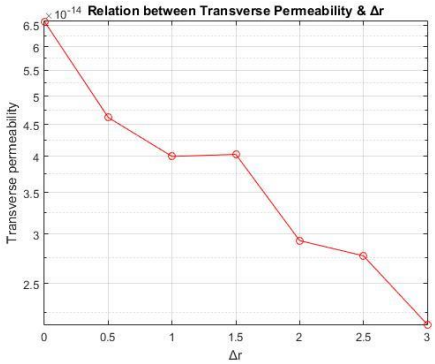


Figure 9: Relation between Transverse Permeability & Δr

(δ_{min} = 0.35 μm, ε = 0.5, L=100 μm)

Minimum inter fiber distance δ_{min}

According to the simulation results, it seems that δ_{min} does not have an noticeable contribution to the permeability, since under the condition of Δr = 0 μm, L = 100 μm, ε = 0.6 and δ_{min} in the range 0.1 μm to 1 μm, the values of permeability are very close (range from 1.90×10^{-13} m^2 to 2.26×10^{-13} m^2). Nevertheless, according to Chen and Yazdchi [8, 9], δ_{min} will have an effective influence on the formation of the micro flow channel and will affect the flow property at the micro scale.

Side length of the domain L

In our model, L is an input parameter which will influence the total number of the fiber since it will directly increase or decrease the domain.

Porosity ε

While the porosity is higher than 0.75, the constant radius of fiber structure will achieve a lower permeability. But if the porosity is higher than 0.75, the obstacles of the larger radii will have an obvious influence on the flow of the fluid while smaller radii have little effects since the ratio of the smaller radii

is low as shown in figure 10.

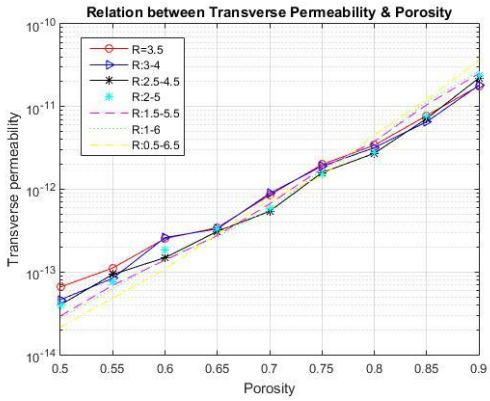


Figure 10: Relation between Transverse Permeability & Porosity
(δ_{min} = 0.35 μm, Δr = 0 μm, L =100 μm)

4 GLOBAL SENSITIVITY ANALYSIS

Morris method [13] and Sobol method are two common methods used to calculate GSA, Sobal method is based on Monte Carlo integral which requires huge amount of calculation. While Morris Method is based on OAT (one-factor-at-a-time), it is relatively high efficiency. Compared with Sobol method, since the simulation cost of our model is high and the sample number is limited, Morris method is more appropriate for this study.

The "Elementary effect" method [14] is based on the concept of building trajectories in a unit hypercube parameter space. It is a local sensitive analysis method while combined with Morris sampling method, it will become a global sensitive analysis method. The i th factor elementary effect can be expressed as the equation below.

$$Ee_i = \frac{y(x_1,\cdots x_i+\Delta,\cdots x_k)-y(x_1,\cdots x_i,\cdots x_k)}{\Delta} \tag{7}$$

where Δ is the variation of x_i, which is a multiple of 1/(p - 1). p is the number of the levels which is normally set to be an even number. It is assumed that Ee_i submits to a certain distribution. Thus, the mean value μ_i^* can describe the effluence of factor x_i on the result y. And the standard deviation σ_i estimates the interaction between the factors.

Morris Sampling matrix is a random sampling method, which keeps the same probability for all the factors. According to Morris method the total calculation times is r*(k+1), here r is the time of sampling. In our model, there are four parameters, k=4, so the total times need to be calculated is 10*(4+1)=50.

Morris sampling method requires to discretize each factor value to a section of [0,1] to keep uniform. Four factors, δ_{min}, Δr , L and ε are discretized to four levels, as shown in table 4.

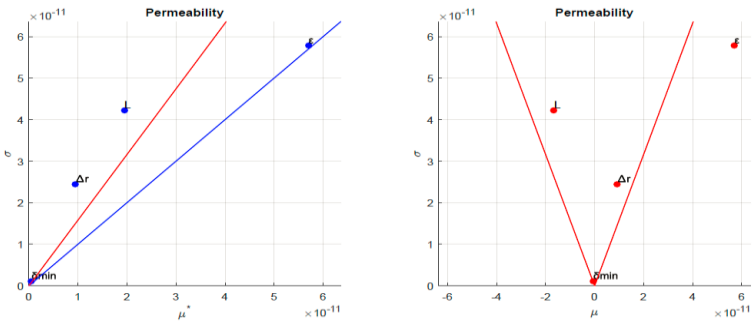
Factors	Level (P=4)			
	0	1/3	2/3	1
δ_{min}(μm)	0.1	0.4	0.7	1
Δr(μm)	0.5	1.333	2.167	3
L(μm)	100	233	367	500
ε	0.5	0.65	0.8	0.95

Table 4: Factors and corresponding levels used to compute the sensitivity

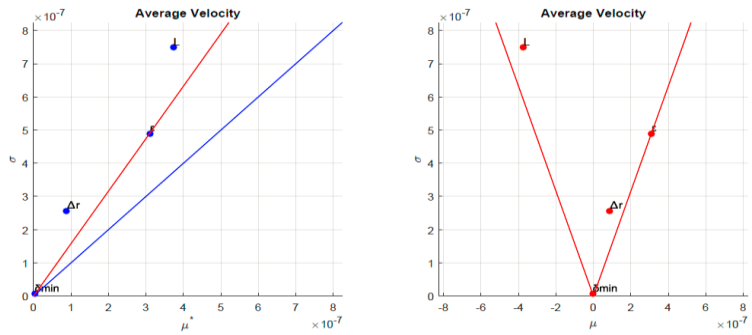
One of the 50 sampling matrices produced by the code is shown below. There is only one different value between each two neighboring lines and the values correspond to the levels of each factor in each column, proving the sampling method here is correct.

$$\begin{bmatrix} \delta_{min} & \Delta r & L & \varepsilon \\ 1 & 3 & 233 & 0.8 \\ 0.4 & 3 & 233 & 0.8 \\ 0.4 & 1.333 & 233 & 0.8 \\ 0.4 & 1.333 & 233 & 0.5 \\ 0.4 & 1.333 & 500 & 0.5 \end{bmatrix}$$

Figure 11 shows the results calculated by Morris Method with 50 groups of data. For the permeability, $\varepsilon > L > \Delta r > \delta_{min}$ and for the average velocity, the relation is $L > \varepsilon > \Delta r > \delta_{min}$. It seems that δ_{min} has the lowest influence degree for both the permeability and the average velocity compared with other three factors. According to [13], the macro factor ε has an obvious influence on the permeability.



(a)



(b)

Figure 11: (a) Morris test results of the Permeability (b) Average Velocity

(Red line on the left: $\mu^* = 2 * \sigma/\sqrt{r}$, blue line: $\mu^* = \sigma$, red lines on the right: $\mu = \pm 2 * \sigma/\sqrt{r}$)

5 CONCLUSION

A random configuration model for fiber arrangement is proposed and implemented in a computer code to calculate the permeability and the average velocity. The influence of 3 micro structural parameters (L, Δr and δ_{min}) and one macro parameter (ε) on permeability and average velocity are studied by GSA Morris method.

According to the analysis, the porosity ε has the most obvious influence on permeability while L has a dominant effect on the average velocity among all parameters. On the other hand, δ_{min} does not show an important effect on the final permeability compared with other factors. The results obtained in this research improved our understanding of the correlation of factors with the permeability. Further

numerical simulation of microscopic and mesoscopic dual scales flow and 3D models can base on some results of this research.

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