ASYMPTOTIC ANALYSIS METHOD FOR HEAT CONDUCTION PROBLEM OF COMPOSITE MATERIALS IN CURVILINEAR COORDINATES

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ABSTRACT

The second-order two-scale (SOTS) asymptotic analysis method is presented for the heat conduction problem concerning composite materials with periodic configuration in curvilinear coordinates. By the asymptotic expansion, the cell problems, effective material coefficients and homogenised heat conduction problems are obtained successively. The main characteristic of the approximate model is that each cell problem defined on the microscopic cell domain is associated with the macroscopic coordinate as a result of the existence of metric function from the coordinate transformation. The error estimation of the second-order expansion is established on a regularity hypothesis. Some common coordinate transformations are discussed and the reduced SOTS solutions are presented. Especially by considering the general one-dimensional problem, the explicit expressions of the SOTS solutions are derived and stronger error estimation is presented. Finally, the corresponding finite element algorithms are presented and numerical results are analysed. The numerical errors presented agree well with the theoretical prediction, which demonstrate the effectiveness of the second-order asymptotic analysis method. By the coordinate transformation, the asymptotic analysis method can be extended to more general domain with periodic microscopic structures.

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