EXPERIMENTAL INVESTIGATION OF FREE OSCILLATION OF CARBON FIBER REINFORCED PLASTICS

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Keywords: carbon fibre, damping behaviour, attenuation factor, oscillation test

ABSTRACT

Within this paper the free oscillations of carbon fiber reinforced plastics (cfrp) are measured by means of a torsion oscillating test. Therefore a new test setup is developed, designed and built. The test setup mostly consists of a cfrp pipe and an oscillating weight, which is a circular disk out of steel. The upper end of the cfrp-specimen is fixed into the torsional load cell and hangs vertically to the ground. On the lower end of the specimen the oscillating weight is fixed. In the course of the test, the oscillating weight is rotated by hand and gets afterwards released. The free oscillations are measured with two different systems. Besides the torsion load cell, an optical measuring system is used to record the test data. In a comparative analysis the strengths and weaknesses of each measuring system are to be identified and are compared with one another. The decrease of moment and displacement resulted during the measurements enable the determination of the attenuation factor through the use of logarithmic decrement.

1 INTRODUCTION

In theory, every technical swinging system exists of separated single spring and damper elements. However, in reality, both properties are combined in the material itself. The stiffness properties of cfrp´s are well known and can be calculated pretty accurate. In contrast, the damping properties are not well known. Both properties are necessary for the design of a technical swinging system. The aim of this paper is the experimental determination of the attenuation factor of a cfrp pipe, to build up a mechanical swinging system based on the measured results.

Therefore, a new test setup is developed, designed and built. The test setup is designed numerically and analytically for a torsion eigenfrequency below ten Hertz. In that way, the optical measuring of the test can be ensured. In order to obtain a low torsion eigenfrequency, a great length to diameter ratio is necessary. This causes that the bending eigenvalue of the setup is close by. Therefrom, the analytical design is used to create a gap between the torsion eigenvalue and the bending eigenvalue as large as possible. Design variables for this difference in eigenfrequencies are as follows: the diameter and thickness of the swinging weight, and the length of the specimen. In that way, by using the optimized design, it should be possible to avoid an overlapping of both swinging eigenmodes.

2 DESIGN OF EXPERIMENTAL SETUP

This chapter describes the design of experiment. The test setup mostly consists of a cfrp pipe and an oscillating weight. The upper end of the cfrp-specimen is fixed to the torsion load cell and hangs vertically to the ground. On the lower end of the specimen the oscillating weight is fixed which a circular disk out of steel is. For the analytical design of the experiment, the cfrp-pipe is simplified as spring element on the one hand and the oscillating disk as a rigid body on the other hand. Only the torsional and bending stiffness of the cfrp pipe and the moment of inertia of the oscillating disk are of interest.

2.1 Analytical Design

For the analytical design of the experiment, the test setup is simplified in figure 1.

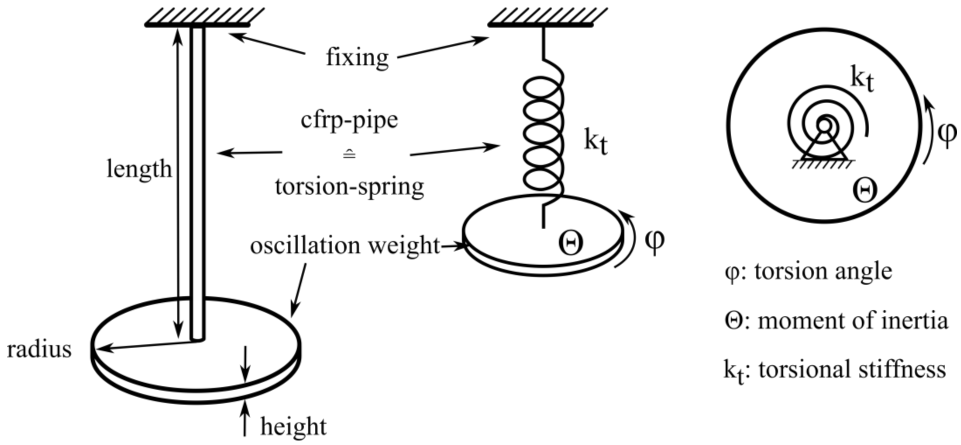


figure 1: Simplified test setup

As described before the cfrp-pipe is modeled as a torsion spring with the torsional stiffness k_t. Based on the mechanical equivalent circuit following differential equation is posted:

$$\Theta\ddot{\varphi} + k_t\varphi = 0 \rightarrow \ddot{\varphi} + \omega_{0,t}^2\varphi = 0 \tag{1}$$

$$\omega_{0,t} = \sqrt{\frac{k_t}{\Theta}} \tag{2}$$

The torsional stiffness and the moment of inertia can be described as follows [2]:

$$\Theta = \frac{1}{2}mr^2 = \frac{1}{2}\pi\rho r^4 h \tag{3}$$

$$k_t = \frac{GI_t}{l} \tag{4}$$

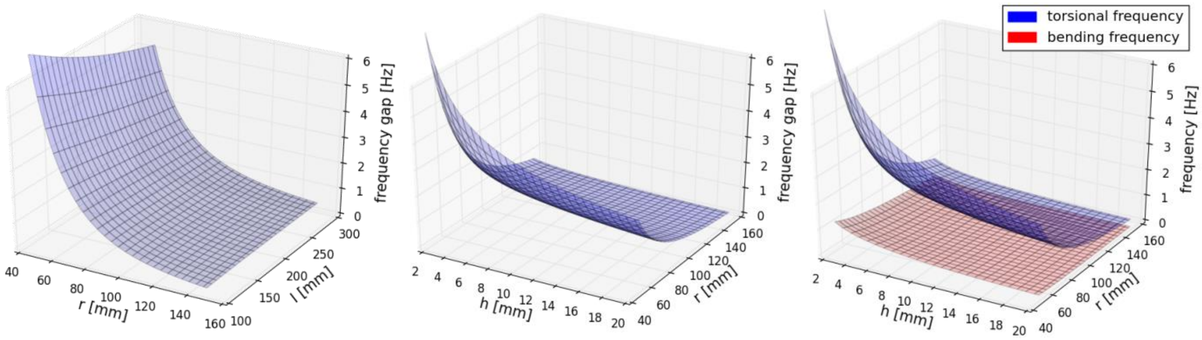
If the equations (3) and (4) are used in equation (2), it follows for the torsion eigenvalue:

$$\omega_{0,t} = \sqrt{\frac{2GI_t}{\pi\rho r^4 hl}} \tag{5}$$

Design variables of the oscillation test are the length of the torsion-spring *(l)*, the radius *(r)* and the height *(h)* of the oscillation weight. As a material for the oscillation weight, steel is defined. Other materials like aluminium would be too light and the torsion eigenvalue would be accordingly too high. The geometry of the cfrp pipe is specified previously. It has an inner diameter of 2.7 mm and an outer diameter of 3.1 mm. The fibre orientation is ±45° to the axial axis of the braided tube. The shear module is calculated by *AlfaLam* [4], which is an analysis tool for laminates. By using the same approach, the eigenfrequency of the nearby bending value is examined:

$$\omega_{0,b} = \sqrt{\frac{12EI_b}{\pi\rho r^2 lh(r^2 + h^2 + 12l^2)}} \tag{6}$$

With the equations (5) and (6) a difference of both eigenfrequencies can be determined. This leads to an equation with four unknown, one of them is the gap between the frequencies, the remaining three are the design variables of the experiment. The aim of the analytical design is to make the difference between the frequencies as large as possible. Therefore the gap between the frequencies and two design variables are plotted (Figure 2), one of the design variables has to be fixed. In Figure 2 (a) on the x-axis the radius *(r)* and on the y-axis the length *(l)* is depicted. The z-axis illustrates the difference of frequencies. The height of the disk is calculated through the radius *(r)* and a defined radius to height *(h)* ratio of 15. This figure shows that the length of the specimen does not have a huge impact on the difference of the frequencies. In consequence, for the following analysis a length of the specimen is fixed to 200 mm.



(a) (b) (c)

Figure 2: Results of the design of experiment

In Figure 2 (b) on the x-axis the height and on the y-axis the radius of the rotating disk is plotted. A clear increase in the difference between the frequencies can be achieved by simultaneously decreasing height and radius of the rotation disk. In Figure 2 (c) both frequencies, the torsional and the bending frequency, are depicted. That is requested for choosing a setup with a torsion frequency below ten Hertz. Unfortunately, the bending frequency is the first eigenfrequency of the test setup and consequently the difference between the frequencies is limited by the optical measuring system. As a result of the before elaborated discussion the geometry listed in Table 1 for the test setup is set.

Design variable		value
h	[mm]	4
r	[mm]	50
l	[mm]	200

Table 1: Design of the test setup.

The analytical calculations of the chosen test setup lead to a torsion eigenvalue of 6.25 Hz. The bending eigenvalue is 0.76 Hz.

2.2 Numerical Design

The analytical design of the experiment is validated by numerical analyses. Therefore, the geometry described in Table 1 is rebuilt in the finite-element-software *Abaqus*®. By making use of a modal analysis, the eigenvalues are calculated. The numerical calculations lead to a torsion eigenvalue of 6.41 Hz. The bending eigenvalue of the test setup is 0.91 Hz. Therefore, the analytical results can be confirmed.

3 EXPERIMENTAL VALIDATION

This chapter focuses on the experimental execution of the free oscillation test. As described before, two measuring methods are used and the results of both will be presented within the framework of this paper. For the test, the disk is rotated by hand and released afterwards. A torsion load cells records the torsional moment throughout the time and the optical measuring systems tracks the displacement of markers on the surface of the disk. The experimental setup is illustrated in Figure 3.

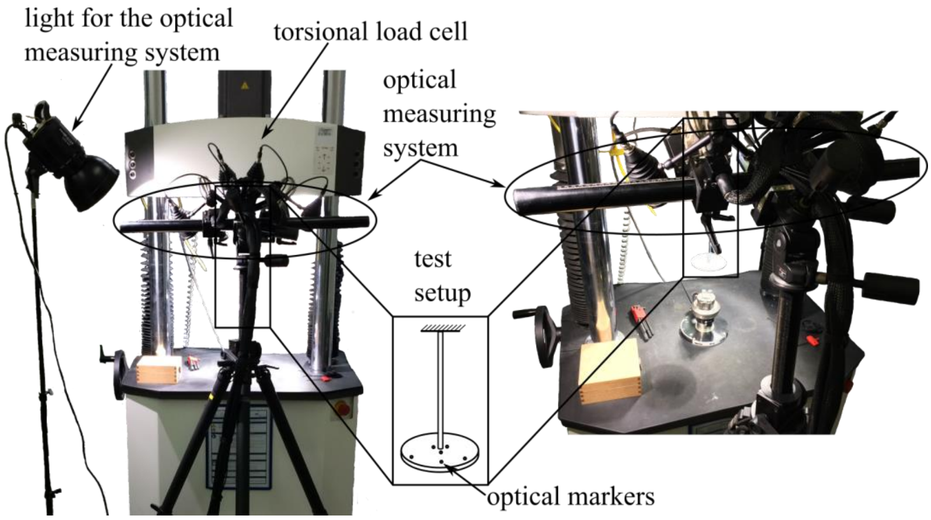


Figure 3: Test setup

3.1 Torsion Load Cell

The results of the torsional load cell are shown in Figure 4 below. The figure is structured in four subfigures which demonstrate the whole process from recording the data to the interpretation. In Figure 4 (a) the raw data is plotted. As the enlargement of the raw data in subfigure (b) demonstrates, a smoothing of the data is necessary in order to receive a clear maximum and minimum of each peak of the curve. In addition, the curve needs to be shifted to the x-axis. This results as a consequence of the hysteresis of the load cell. The decrease of maxima and increase of minima can be used for calculating the logarithmic decrement only by using a shifted curve. The final Figure 4 (d) presents the smoothed and shifted data of one free oscillating test.

The logarithmic decrement is used for calculating the attenuation factor. This is done by following equation [1]:

$$\Lambda = \frac{1}{m-n} ln \frac{\hat{q}_n}{\hat{q}_m} \tag{7}$$

$$D = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}} \tag{8}$$

In order to calculate the attenuation factor, only neighbouring maxima or minima are analysed. In total 13 tests were analysed with the torsion load cell. The development of the attenuation factor is also shown in Figure 4 (d). It is recognizable that the attenuation factor changes throughout the time of the free oscillation. Further details on this subject are described and discussed in chapter 4. The frequency of the torsional swinging is analysed by a fast fourier transformation (fft), with a value of 4.09 Hz. The calculated attenuation factor of all analysed test amounts 1.3 %.

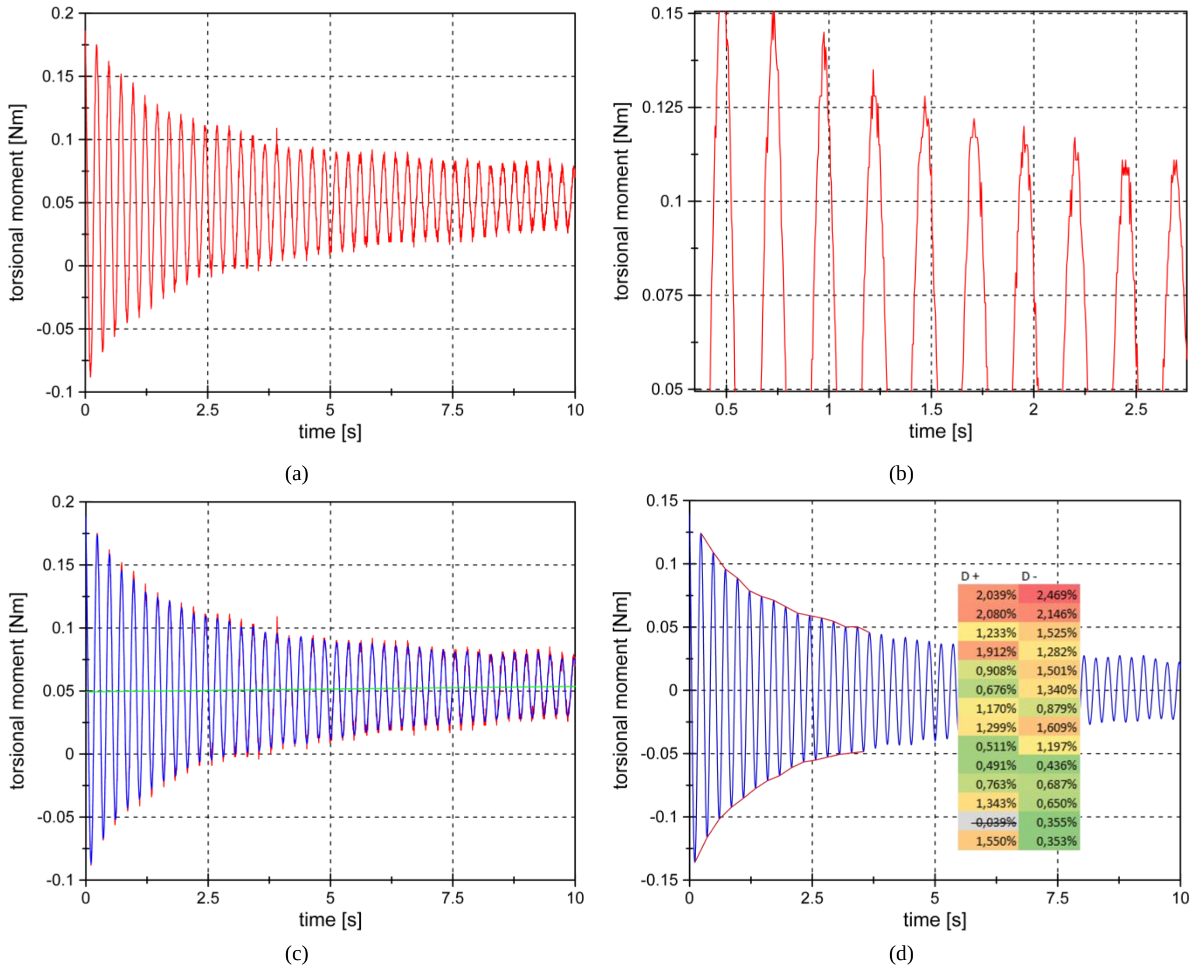


Figure 4: Experimental evaluation of the torsion load cell

3.2 Optical Measuring System

Despite of the optimization of the geometry which created a gap as largely as possible between the eigenfrequencies, test execution illustrates that there is an overlap of eigenmodes. Figure 5 shows the complete process of the recorded data and the separation of eigenmodes. For recording the torsional oscillation, the displacement of a point of the disk has to be tracked. The displacement of the midpoint is essential for separating the bending and torsion eigenvalue. In a pure torsional oscillation the midpoint of the disk has no movement. Therefore Figure 5 (a) is an indication for the modulation of frequencies. An fft of the midpoint displays only one frequency is active in the displacement of the midpoint, this is the bending frequency. Tracking of the displacement of a point on the outside shows a modulation frequency (Figure 5 (c)). As suggested, it is a modulation of two frequencies. This is displayed by another fft (Figure 5 (d)). One of them is the bending and the other one the torsion frequency. In theory eigenmodes are orthogonal to each other and can be separated [1]. For this purpose, the midpoint movement is subtracted from the movement of the points on the outside. The result of this difference is plotted in Figure 5 (e), as an fft of the movement demonstrates; only the torsion frequency is left and the curve in Figure 5 (e) is used for calculating the attenuation factor.

Similar to the analysis with the torsion load cell, only neighbouring maxima or minima are used for calculating the attenuation factor. In total three tests are performed and with each test four points are analysed with the optical measuring system. The development of the attenuation factor is also plotted in Figure 5 (e). It appears that the attenuation factor changes throughout the time of the free oscillation. The discussion of this subject is elaborated in detail once again in chapter 4. The frequency of the torsional swinging is analysed by an fft, the value amounts 4.32 Hz, whereby the value of the nearby bending eigenfrequency is 1.35 Hz. The averaged attenuation factor of the optical measuring is 0.75%.

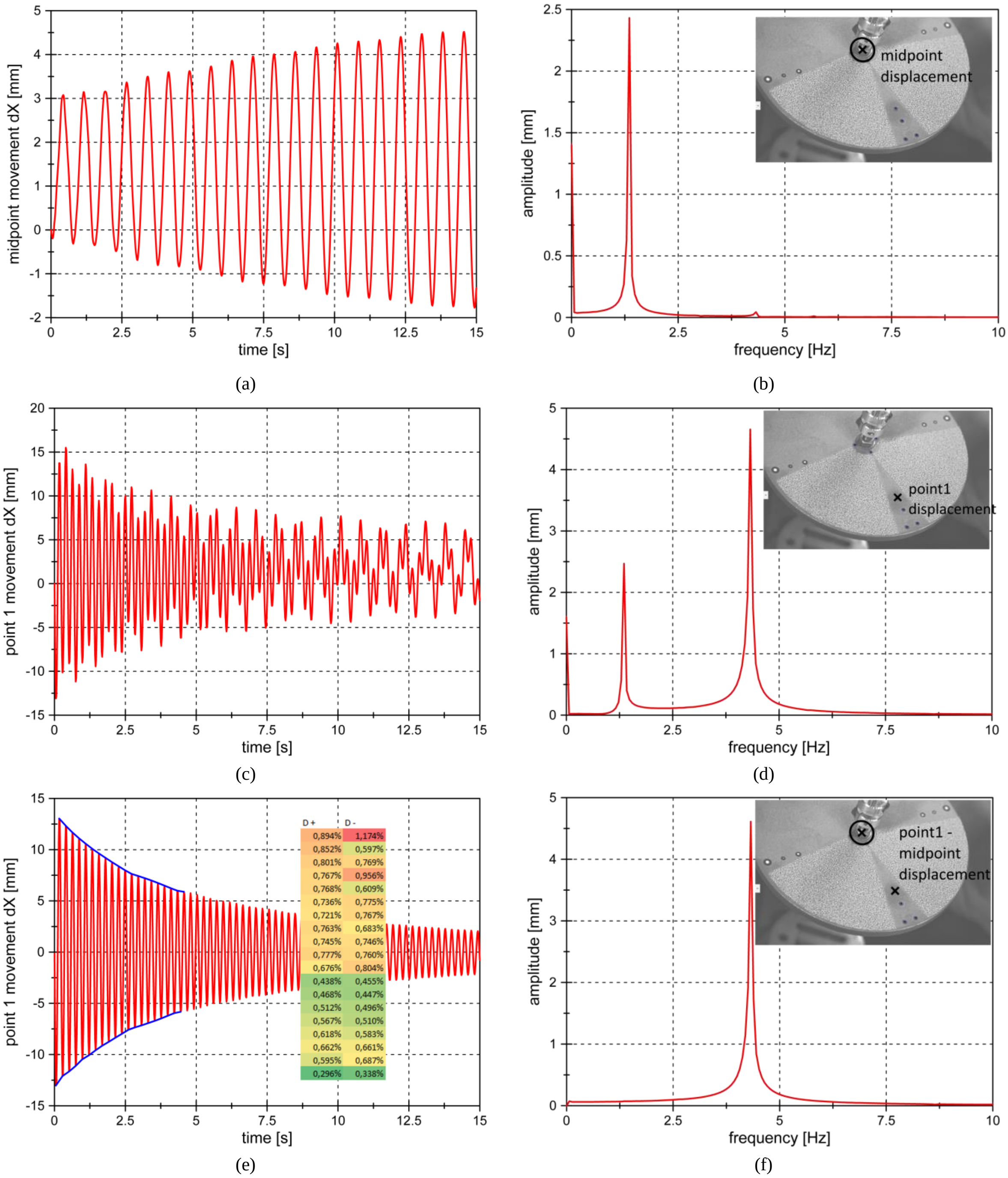


Figure 5: Experimental evaluation of the optical measuring system

4 DISCUSSION

Among the torsion frequency determined by the torsion load cell and the optical measuring system the difference is relatively small. In contrast, the difference in the attenuation factor deviates slightly and is little higher. This is based on the measuring range of the torsion load cell. The range is up to 70

Nm, although the measuring of the oscillation test only rises to torsional moment of 0.2 Nm. An upside of the torsion load cell is the fast and uncomplicated evaluation of the test data. Even though there is a modulation of frequencies, the torsion load only records the data of the torsional oscillation and it can be stated that the bending oscillation has no influence on the record.

The difference of the torsion eigenfrequency of the design of experiment and the actual measured frequency in the test is quite big. Several influences on the test setup are responsible for this. First of all, the design in chapter 2 is a simplified design. The contact of the different parts of the setup is modelled perfectly stiff and with no friction between the parts. Secondly, the design does not consider any damping behaviour. In the test setup, there is friction among the parts. In consequence, this leads on the one hand to a loss of stiffness, and on the other hand to contact damping. Besides, the contact damping air friction is an additional factor. Both parameters eventuate in a lower frequency of the test.

The decreasing of the attenuation factor throughout a test can also be explained by air friction. Air friction, and accordingly the damping, depends on the velocity of the rotating disk [3]. During the test, the speed of the rotating disk decreases and consequently the attenuation decreases.

5 CONCLUSIONS

Within this paper a free oscillation test for cfrp pipes is designed, built and evaluated. The analytical design tries to separate the eigenmodes as far as possible, but test execution displays otherwise. With the chosen test setup a modulation of eigenmodes is not avoidable.

Both measuring methods, the torsional load cell and the optical system, can be used for test recording. The torsional load cell allows a fast and uncomplicated evaluation of the test data but the range of the cell is not optimal for the test. An optical measuring of the test data is possible, but analysis of the test results requires extra effort.

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