INTERNAL DEFORMATION OF A SANDWICH PLATE
MAPPED BY DIGITAL VOLUMETRIC SPECKLE PHOTOGRAPHY

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ABSTRACT
Composite materials have vital applications in many areas. However, detailed internal deformation patterns are not easily to obtain using conventional experimental methods. In this paper, we apply a 3D strain analysis technique called Digital Volumetric Speckle Photography (DVSP) to calculate the internal deformation of a sandwich plate under three-point bending experiment and show some displacement and strain fields distributions along different sections. According to the results, DVSP is an effective way to map the internal stain and will be an important method in measurement field.

INTRODUCTION
Sandwich plates or shells are ubiquitous in ship construction when composite material is used. The sandwich consists of two face sheets made of a more stiff material such as fiber reinforced composite and a core made of a softer material such as foam. Such a structure has a high bending stiffness but also low weight thus giving rise to a high strength to weight ratio. The stress/strain distribution of a sandwich plate under load is rather complex. Damage usually results in delamination of face sheet(s) and shear cracks of the core. Detailed internal deformation patterns are not obtainable using conventional experimental techniques. In this paper we show some results of the internal deformation of a sandwich plate under three-point bending by applying a newly developed 3D strain analysis technique called Digital Volumetric Speckle Photography (DVSP)[1-3].

THEORY OF DVSP
Digital volume images of a 3D solid before and after deformation are reconstructed from a Micro-CT recording. These two volume images are defined as reference volume image and deformed volume image, respectively. They are then divided into subsets with voxel arrays of 32×32×32 voxels, for example, and ‘compared’. The result is a displacement vector
representing the collective movement experienced by all the speckles within the subset. Based on the theory of 2D digital speckle photography technique, the DVSP principle is described as follows:

Let \( h_1(x,y,z) \) and \( h_2(x,y,z) \) be the complex amplitudes of the light disturbance of a generic speckle subset pair, before and after deformation, respectively, and

\[
\begin{align*}
  h_1(x, y, z) & = h\left[x - u(x, y, z), y - v(x, y, z), z - w(x, y, z)\right] \\
  h_2(x, y, z) & = h\left[x - u(x, y, z), y - v(x, y, z), z - w(x, y, z)\right]
\end{align*}
\]  

(1)

Figure 1. Schematics demonstrating the processing algorithm of DVSP

where \( u, v \) and \( w \) are the displacement components experienced by the speckles along the \( x \), \( y \) and \( z \) directions respectively. A first-step 3D FFT is applied to both \( h_1 \) and \( h_2 \) yielding

\[
\begin{align*}
  H_1(f_x, f_y, f_z) & = \mathcal{F}\{h_1(x, y, z)\} \\
  H_2(f_x, f_y, f_z) & = \mathcal{F}\{h_2(x, y, z)\}
\end{align*}
\]  

(2)

where \( H_1(f_x, f_y, f_z) \) is the Fourier transform of \( h_1(x, y, z) \), \( H_2(f_x, f_y, f_z) \) is the Fourier transform of \( h_2(x, y, z) \) and \( \mathcal{F} \) stands for Fourier Transform.

Then, a numerical interference between the two 3D speckle patterns is performed at the spectral domain, i.e.

\[
F(f_x, f_y, f_z) = \frac{H_1(f_x, f_y, f_z)H_2^*(f_x, f_y, f_z)}{\sqrt{|H_1(f_x, f_y, f_z)|^2}}
\]

\[
= |H_1(f_x, f_y, f_z)|\exp\left[i\Phi_1(f_x, f_y, f_z) - \Phi_2(f_x, f_y, f_z)\right]
\]  

(3)

where \( \Phi_1(f_x, f_y, f_z) \) and \( \Phi_2(f_x, f_y, f_z) \) are the phases of \( H_1(f_x, f_y, f_z) \) and \( H_2(f_x, f_y, f_z) \) respectively. And * stands for the complex conjugate. It is seen that

\[
\Phi_1(f_x, f_y, f_z) - \Phi_2(f_x, f_y, f_z) = 2\pi uf_x + vf_y + wf_z
\]  

(4)

Finally, the function is obtained by performing another 3D FFT resulting
which is an expanded impulse function located at \((u, v, w)\). This process is carried out for every corresponding pair of subsets. By detecting the crest of this impulse function, an array of displacement vectors at each and every subset are obtained, from which strain tensors can be calculated using an appropriate strain-displacement relation.

Based on this approach, the deformation of the subset itself is neglected. Because of the discrete nature of digital volume images, the displacements evaluated from equation (5) are integer multiples of one voxel. In order to obtain more accurate and sensitive characterization, a sub-voxel investigation of the crest position is desired. Surrounding the integer voxel of the crest, a cubic subset with size of 3×3×3 voxels is selected, a spline interpolation is employed to obtain the sub-voxel accuracy in this work.

A SANDWICH BEAM UNDER THREE-POINT BENDING

Fig.2 shows the sandwich beam specimen under three-point bending. The specimen is made of E-glass Vinyl Ester composite face sheet with a polymer foam core [4]. The size is 50.0×20.0×32.0 mm³. The loading history is plotted as shown in Fig.3. The whole loading process is divided into 6 steps. In each step, the load is kept constant, and the specimen is scanned by using Micro-focus X-ray Computed Tomography (CT) shown, and then serial slice images are obtained. The CT system is shown in Fig.4. The reconstructed volumetric images have 890×360×530 voxels with the physical size of a voxel being 55×55×55 μm³.

![Fig.2](image_url) (a) Specimen, (b) X-ray projected image of the specimen

![Fig.3](image_url) Loading-deflection curve
DVSP was employed to analyze the 3D deformation field of seven internal sections of the foam core under the progressively increasing load. The size of the calculated region is 49.0×19.8×23 mm³, whereas the size of a subset is 32×32×32 voxels. The volumetric image of the first step is defined as the reference volumetric image, and the subsequent images as the deformed volumetric images. The results of loading step 5 (1800N) are selected to demonstrate the capability of DVSP. The interior displacement field of five transverse sections and two longitudinal sections are depicted in the Fig.5.
From 3D displacement fields, the internal strain field can be calculated. Here, we use the point-wise least-squares (PLS) approach to calculate the strain fields, the results of which are shown in Fig. 6.

CONCLUSION

DVSP (Digital Volumetric Speckle Photography) has been successfully applied to mapping the internal strain distribution in a sandwich beam under three-point bent. The heterogeneous micro-structures displayed in different gray values in the CT image of the beam were treated as 3D speckles. The movements of these volumetric speckles under load are tracked by the DVSP algorithm. We believe this technique will, in the near future, contribute to the understanding of the 3D nature of the deformation of sandwich structures.
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