FULL 3D INTERIOR DEFORMATION OF A COMPOSITE BEAM WITH PREPARED SLOT UNDER 3-POINT BENDING USING DIGITAL VOLUMETRIC SPECKLE PHOTOGRAPHY AND MICRO-FOCUS X-RAY COMPUTED TOMOGRAPHY

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ABSTRACT

The interior 3D deformation field of a composite beam with prepared slot under three-point bending has been mapped using Digital Volumetric Speckle Photography (DVSP) with subvoxel shifting. The technique employs a Micro X-ray CT system to record a volume image of the specimen and treats the composite beam’s internal texture as 3D volumetric speckles. By extending the algorithm used in 2D speckle photography into 3D we are able to calculate the internal strain field quantitatively. Full field maps of u, v and w are obtained which allow the computation of strain fields. We observed the strain distribution of different sections. According to the results, there are higher strain values in strain fields corresponding to zone of the cracks, which is a good predictor for the impending failure of the beam. Therefore DVSP with subvoxel shifting algorithm will be an effective method to probe the interior deformation of composite beams under mechanical load.

INTRODUCTION

Composite materials, be it fiber reinforced, particle reinforced, woven, etc., can reduce both the weight and manufacturing cost of advanced composite structures such as aircraft, naval vessels, and blades of wind turbines. Many existing works based on direct microscopic observations have identified the degradation mechanisms involved matrix cracking, fiber failures, fiber-matrix debonding or delamination. The experimental measurement of strain has always been an important topic in the investigation of composite material behavior and properties. For advantages of being non-contacting, full-field, simple system, and direct sensing, the technique of Digital image correlation (DIC) has been applied to the measurement of displacements and strains in composites extensively in recent years [1]. However, only the surface deformation of a specimen can be obtained by using DIC. Due to the anisotropic nature of composites, it is difficult to fully predict their mechanical characteristics and behavior just based on the surface deformation. Combined Computed Tomography (CT), Digital Volume Correlation (DVC), the 3D extension of DIC, was proposed to map the interior deformation of composites [2]. Digital Speckle Photography is another 2D deformation measurement technique which was developed independently and in parallel with DIC. DSP is the result of the natural evolution of the Optical Speckle Photography technique which has its basis in Fourier Optics. By taking advantage of FFT, a much higher computational efficiency is achieved by using DSP. As the 3D extension of DSP, Digital Volumetric Speckle Photography (DVSP) has been applied to various materials including composite [3,4]. In this paper show some results by applying DVSP with subvoxel shifting to map the internal strain deformation of a woven composite beam with prepared slot under 3-point bending.
THEORY OF DVSP WITH SUBVOXEL SHIFTING

Digital volume images of a 3D solid before and after deformation are reconstructed with advanced imaging techniques using a CT. These two volume images are defined as reference volume image and deformed volume image, respectively. Both of them are divided into volumetric subsets with voxel arrays of 32×32×32 voxels, for example, and ‘compared’. The principle is schematically shown in Fig. 1. The DVSP principle is as follows:

Let \( h_1(x, y, z) \) and \( h_2(x, y, z) \) be the gray intensity functions of a pair of generic volumetric speckle subsets, before and after deformation, respectively, and that

\[
\begin{align*}
    h_1(x, y, z) &= h(x, y, z) \\
    h_2(x, y, z) &= h[x - u(x, y, z), y - v(x, y, z), z - w(x, y, z)]
\end{align*}
\]

where \( u, v \) and \( w \) are the displacement components experienced by the speckles along the \( x, y, \) and \( z \) directions, respectively. A first-step 3D FFT (Fast Fourier Transform) is applied to both \( h_1 \) and \( h_2 \) yielding

\[
\begin{align*}
    H_1(f_x, f_y, f_z) &= \mathfrak{F}h_1(x, y, z) = |H(f_x, f_y, f_z)| \exp\left[j\phi(f_x, f_y, f_z)\right] \\
    H_2(f_x, f_y, f_z) &= \mathfrak{F}h_2(x, y, z) = |H(f_x, f_y, f_z)| \exp\left[j\phi(f_x, f_y, f_z) - 2\pi(u f_x + v f_y + w f_z)\right]
\end{align*}
\]

where \( H_1(f_x, f_y, f_z) \) is the Fourier transform of \( h_1(x, y, z) \), \( H_2(f_x, f_y, f_z) \) is the Fourier transform of \( h_2(x, y, z) \), and \( \mathfrak{F} \) stands for Fourier Transform. \( |H(f_x, f_y, f_z)| \) and \( \phi(f_x, f_y, f_z) \) are spectral amplitude and phase fields, respectively.

Then, a numerical interference between the two 3D speckle patterns is performed at the spectral domain, i.e.

\[
F(f_x, f_y, f_z) = \frac{H_1(f_x, f_y, f_z)H_2^*(f_x, f_y, f_z)}{\sqrt{|H_1(f_x, f_y, f_z)H_2(f_x, f_y, f_z)|}} = |H_1(f_x, f_y, f_z)| \exp\left[j\phi(f_x, f_y, f_z) - \phi_2(f_x, f_y, f_z)\right]
\]
where \( \phi \left( f_x, f_y, f_z \right) \) and \( \phi \left( f_x, f_y, f_z \right) \), are the phases of \( H \left( f_x, f_y, f_z \right) \) and \( H \left( f_x, f_y, f_z \right) \), respectively. It is seen that

\[
\phi \left( f_x, f_y, f_z \right) - \phi \left( f_x, f_y, f_z \right) = 2\pi \left( uf_x + vf_y + wf_z \right)
\]

Finally, a function is obtained by performing another 3D FFT resulting

\[
G(\xi, \eta, \zeta) = \mathcal{F} \left[ f_x, f_y, f_z \right] = \mathcal{F} \left[ f_x, f_y, f_z \right] = \mathcal{F} \left[ f_x - u, f_y - v, f_z - w \right]
\]

which is an expanded impulse function located at \((u, v, w)\). This process is carried out for every corresponding pair of the subsets. By detecting the crest of all these impulse functions, an array of displacement vectors at each and every subset is obtained.

Because of the discrete nature of digital volume images, the displacement vectors evaluated from equation (7) are integer multiples of one voxel. We selected a cubic subset with 3x3x3 voxels surrounding an integer voxel of the crest and a cubic spline interpolation is employed to obtain the sub-voxel accuracy.

In the DVSP method, if the displacement between the subsets is large, a fair amount of noise is introduced in the correlation surface resulting from the nonoverlapping areas. And this effect gives rise to a poor signal to noise ratio and results in worse measurement performance. The essence of subvoxel shifting algorithm is to minimize the nonoverlapping area by shifting one of the subsets for noninteger voxel values until a maximum fit is obtained between the two subsets[5]. One subset from the reference volumetric image and the corresponding subset from the deformed volumetric image are then “compared” via the 3D FFT algorithm. By way of DVSP we can calculate the displacement vector of the subset. If the displacement is found to be larger than one voxel in any direction, a new deformed subset is chosen corresponding to the estimated displacement in integral voxels. The new deformed subset is shifted for nonintegral voxels along \(x\), \(y\), and \(z\) directions, respectively, by using the technique of Fourier shifting. And cross-correlation calculation is then performed until a maximum fit is obtained between the two subset. If the displacement is found to be no more than one voxel in any direction, the original deformed subset is shifted in a nonintegral voxel fashion along \(x\), \(y\), and \(z\) directions, respectively, by Fourier shifting. A cross-correlation calculation is again performed to find a maximum fit. A subvoxel correction of the displacement along three directions is obtained by determining the maximum of a spline or cubic interpolation of the correlation factors. The interpolation is performed by considering the maximum voxel and its neighbors. DVSP with subvoxel shifting is effective in reducing the error by one order of magnitude in general.

**INTERIOR MEASUREMENT OF COMPOSITES DEFORMATION**

The composite material is from the blade of a windmill. The 3-point bending experiment of a specimen with prepared slot was conducted. The dimension of the specimen is 39 mm \(\times\) 18 mm \(\times\) 8.5 mm, and the size of the slot is 3.60 mm \(\times\) 0.68 mm. The load was applied incrementally in 10 steps. At each and every load step, the specimen was scanned using a Micro-CT, and the volumetric image was reconstructed. The size of volumetric image was 960 \(\times\) 424 \(\times\) 260 voxel, and the size of one voxel is 45 \(\times\) 45 \(\times\) 45 \(\mu\)m

The mid-section of the specimen was selected to be analyzed. Fig.1 (a) and (b) displayed the 3D image along the mid-section at the last load step (Step 9 with load being 7.40 kN) before failure and after failure (Step 10). From Fig.1 (a), it can be seen that macro cracks occur in the upper left and the tip of the slot. But from Fig.1 (a), there was no visible sign of impending failure.

When applying the DVSP with subvoxel shifting algorithm to composite materials the first issue is whether or not the Micro-CT (Computed Tomography) can capture the minute interior details of the composite material. From Fig.1, it can be seen that all the details of the composite have been captured. The marks, defects, fiber boundaries can all serve as 3D speckles and be used for calculation. The volumetric image of step 5 (load at 5.0 kN) was kept as the reference image and the one of step 9 as
the deformed image. By applying the algorithm, the 3D displacement field \( u, v \) and \( w \) at step 9 were calculated, and the displacement field of the mid-section is displayed in Fig.2. The subset size had \( 32 \times 32 \times 32 \) voxel. From 3D displacement fields, the internal strain field can then be estimated.

![Fig.1 3D reconstructed image](image1)

(a) Before failure
(b) After failure

![Fig.2 3D displacement (u,v,w) distributions of the mid-section](image2)

(a) mid-section (a) \( u \) field; (b) \( v \) field; (c) \( w \) field
Fig. 3 3D strain distributions of the mid-section (a) $\varepsilon_{xx}$ fields; (b) $\varepsilon_{yy}$ fields; (c) $\varepsilon_{zz}$ fields.

The normal strain fields are showed in the Fig.3. From Fig.2(b) and Fig.3(a), the periodic structure of the specimen results in periodic patterns in displacement and strain distributions. In Fig.4, different section images near the slot, displacement fields and strain fields corresponding to these images are shown. From Fig. 4, strain concentration as clearly indicated in various locations. Corresponding to zone of the cracks, there are higher strain values in $\varepsilon_{yy}$ fields as shown in Figs.4(j)-(l).
Fig. 4 Displacement distributions near the slot in different sections (a) section along \(z=1.9\text{mm}\) (b) section along \(z=4.1\text{mm}\) (c) section along \(z=6.0\text{mm}\) (d) u field of (a) (e) u field of (b) (f) u field of (c) (g) v field of (a) (h) v field of (b) (i) v field of (c) (j) \(\varepsilon_{yy}\) field of (a) (k) \(\varepsilon_{yy}\) field of (b) (l) \(\varepsilon_{yy}\) field of (c)

CONCLUSION

In this paper we have demonstrated that DVSP with subvoxel shifting algorithm can be effectively applied to probing the interior deformation of composite beams under mechanical load.

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