FAILURE ANALYSIS OF COMPOSITE MATERIALS SUBJECTED TO THERMAL STRESSES: A TRANS-SCALE APPROACH

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Keywords: Representative volume element, Matrix failure, Trans-Scale method, Finite elements

ABSTRACT

Fiber reinforced composite materials and structures have been increasingly adopted in the aviation and aerospace industries, for instance, horizontal and vertical tails for aircrafts and rocket fairings for spacecrafts, due to their superior specific stiffness and strength. For some applications, such as composite over-wrapped pressure vessels (COPVs), the composite materials may be recommended to work at the liquid hydrogen (LH2) temperature and must be able to ensure safety and reliability in service. In this paper, Trans-Scale failure predictions considering micro-stress fields induced by temperature are developed based on representative volume element (RVE). The matrix failure criteria are developed based on LaRC03 criteria and the Puck's Action Plane proposal and the prediction accuracy of the Trans-Scale method is demonstrated.

1 INTRODUCTION

Fiber reinforced composite materials and structures have been increasingly adopted in the aviation and aerospace industries due to their superior specific stiffness and strength [1]. However, the mechanisms of the failure incubation in composite materials are not yet fully understood. The results of the World-Wide Failure Exercise (WWFE) [2–6] indicates that although numerous failure theories have been developed, the predictions from these established failure models and criteria still varies significantly from the experimental observations under different conditions. The homogeneous hypothesis of composite layers has been proven to ignore the internal stresses between fiber and matrix. Thermal stress was found to alter the microscopic stress state significantly [7]. For a thermal stress problem, the differences in thermal expansion coefficients between the reinforcing phase and the matrix phase could result in innegligible internal stresses due to the mismatch between the matrix and fibers. Under some extreme conditions, thermal stress caused by the mismatch of thermal contraction between the fibers and matrix can lead to the initial of micro-cracks [8,9].

Hence, the aim of this paper is to collectively investigate the effects of thermal stress on the transverse fracture behaviour of matrix in fibre reinforced composites. The Trans-Scale computational framework used to model deformation in the carbon fibre/epoxy composite is similar to that established by Ren and co-workers [9]. In a first step, the representative volume element (RVE) is established based on the fiber distribution and the periodic boundary condition and the finite element analysis of RVE is presented to obtain the micro-stress fields. Then the micro failure prediction is demonstrated using the proposed micro failure criteria. Results are discussed and compared with the simulated results published in literature.

2 THE IMPLENTMENT OF THE TRANS-SCALE METHOD

2.1 Micro scale modeling

The shape of RVE is related to the fiber distribution and can be divided into two kinds: regular distribution or random distribution. The regular distribution assumes the fibers are evenly distributed. Random distribution [7] assumes the fibers distribute in matrix following a random function, such as Weibull distribution, or from experimental observation. Compared to the random distribution, the regular distribution significantly reduced computational costs and can also provide satisfying results. Hence, the fibers are assumed to be periodically distributed in this study. Fig.1 shows the cross-section of the unidirectional fiber reinforced composites, and the possible selections of RVEs, square (S1) or hexagonal (H1). It has been proven that the hexagonal RVE H1 is more advanced to represent real materials comparing to square layout approach [10]. Therefore, the hexagonal RVE shown in Fig.1 is adopted in this study.

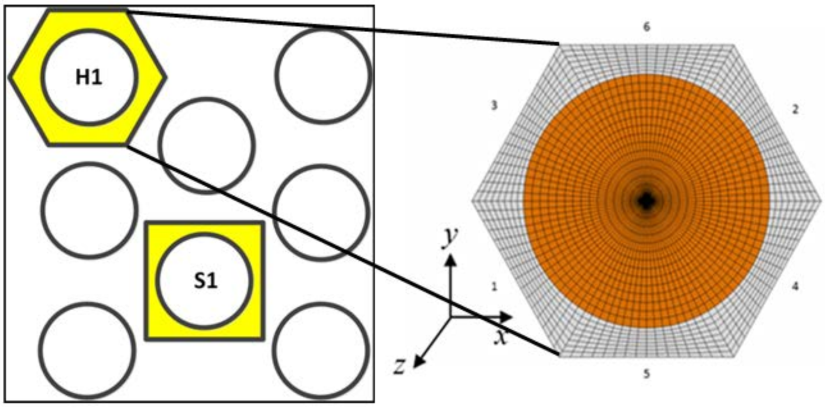

Fig.1 the possible selections of RVEs and the hexagonal RVE employed

Periodic boundary conditions are applied on the RVE to ensure the periodicity of stress/strain fields. The boundary conditions of the hexagonal RVE are designed and provided in Tab.1. The first 6 cases consider 6 uniform strain components and the last case consider uniform temperature change. The details of corresponding formula derivations can refer to Refs.[8]

$\varepsilon_x = 1$	$\varepsilon_y = 1$	$\varepsilon_z = 1$	$\gamma_{xy} = 1$	$\gamma_{xz} = 1$	$\gamma_{yz} = 1$	$\Delta T = 1$
$u_2 - u_1 = \sqrt{3}L/2$	$u_2 - u_1 = 0$	$u_2 - u_1 = 0$	$u_2 - u_1 = L/2$	$v_2 - v_1 = 0$	$u_2 - u_1 = 0$	$u_2 - u_1 = 0$
$u_4 - u_3 = \sqrt{3}L/2$	$u_4 - u_3 = 0$	$u_4 - u_3 = 0$	$u_4 - u_3 = -L/2$	$v_4 - v_3 = 0$	$u_4 - u_3 = 0$	$u_4 - u_3 = 0$
$v_2 - v_1 = 0$	$v_2 - v_1 = L/2$	$v_2 - v_1 = 0$	$v_2 - v_1 = 0$	$w_2 - w_1 = \sqrt{3}L/2$	$w_2 - w_1 = L/2$	$v_2 - v_1 = 0$
$v_4 - v_3 = 0$	$v_4 - v_3 = -L/2$	$v_3 - v_4 = 0$	$v_3 - v_4 = 0$	$w_4 - w_3 = \sqrt{3}L/2$	$w_4 - w_3 = -L/2$	$v_4 - v_3 = 0$
$v_5 = 0$	$v_5 = -L/2$	$v_5 = 0$	$v_5 = -L/2$	$v_5 = 0$	$w_5 = -L/2$	$u_6 - u_5 = 0$
$v_6 = 0$	$v_6 = L/2$	$v_6 = 0$	$v_6 = L/2$	$v_6 = 0$	$w_6 = L/2$	$u_6 - u_5 = 0$
$w_e - w_i = 0$	$w_e - w_i = 0$	$w_e - w_i = t$	$w_e - w_i = 0$	$u_e - u_i = 0$	$v_e - v_i = 0$	$w_e - w_i = 0$

Tab.1 Boundary conditions of the hexagonal unit cell (contains 7 cases)

2.2 Micro failure prediction

As to prediction the matrix failure, the following criteria are proposed based on the LaRC03 failure criteria after the micro stress fields have been obtained. The LaRC03 failure criteria for matrix compression failure are based on a Mohr-Coulomb interaction of the stresses associated with the plane of fracture (see Refs.[11-13]) .

Consider a continuous fiber composite material where the matrix (m) and fibers (f) are allowed to retain their identity in the continuum, the failure envelopes in any coupled stress plane have the form shown in Fig.2. The macro envelope is the inner envelope of the fiber and matrix.

For the matrix and fiber phases, the micro failure yields:

$$MicroFI_m = f(\sigma_m(x, y)) \le 1 \tag{1}$$

$$MicroFI_f = f(\sigma_f(x, y)) \le 1 \tag{2}$$

Here, *x* and *y* refer to the coordinate on the micro scale. σ_m refers to the stress in the matrix and σ_f refers to the stress in the fiber.

The macro failure yields:

$$MacroFI = f(\sigma_i, \sigma_j) \leq 1 \tag{3}$$

Here, σ_i and σ_j are the macro stresses, which are actually the volume average of all micro-stress in both matrix and fiber phases. It should be noted that, if the composite material is supposed to have matrix failure mode, the failure criterion should not have any connection with stress in the fiber. Since then, the macro criteria should have a micro equivalent expression. Consider the failure mechanism that the matrix failure is induced by the effective stresses on the failure plane in the matrix, we have the similar form with [11]

$$\text{Matrix \#01: } FI_M = \left(\frac{\tau_{eff}^T}{S_{mT}}\right)^2 + \left(\frac{\tau_{eff}^L}{S_{mL}}\right)^2 \leq 1 \text{ while } \sigma_{22}^m \leq 0 \tag{4}$$

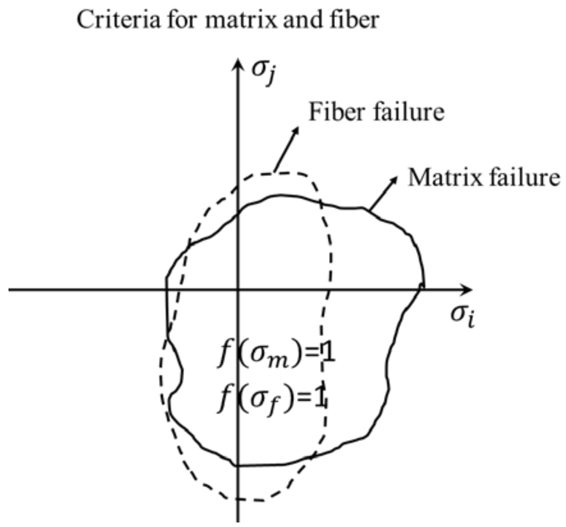

Fig.2 the failure envelopes of fiber, matrix

Different with [11,12], here S_{mT} and S_{mL} are the transverse and longitudinal shear strengths of the matrix, τ_{eff}^T and τ_{eff}^L are the effective stresses on the failure plane defined as:

$$\tau_{eff}^T = \left\langle -\sigma_{22}^m \cos\alpha \left(\sin\alpha - \eta^T \cos\alpha\right)\right\rangle \tag{5}$$

$$\tau_{eff}^L = \left\langle \cos\alpha \left(\left|\tau_{12}^m\right| + \eta^L \sigma_{22}^m \cos\alpha\right)\right\rangle \tag{6}$$

Here, the subscript 'm' refers to stress in the matrix. The matrix #01 failure index has the similar form with LaRC03#01 and based on the same assumption that the failure occurs on the Puck's Action Plane. The stress components are the matrix stresses calculated through the finite element analysis of the RVE, rather than volume average of both fiber and matrix micro-stress.

Note that the the macro tension strength Y_T, is always much smaller than Y_{mT}. This may due to the initial defect such as the initial slit crack assumed by [11]. However, it is difficult for both macro or micro modeling to measure this stress concentration factor because the lack of corresponding information. while macro failure occurs under pure tension, $\sigma = Y_T$, the matrix failure index should also reach 1, which leads to: $\sigma_m = Y_{mT}$ So the stress concentration factor k is defined as:

$$k = \frac{Y_T}{Y_{mT}} \tag{7}$$

For the condition that $\sigma_{22}^m \geq 0$, we have the similar form with LaRC03#02,

$$\text{Matrix \#02: } FI_M = (1-g)\frac{\sigma_{22}^m}{kY_{mT}} + g\left(\frac{\sigma_{22}^m}{kY_{mT}}\right)^2 + \left(\frac{\tau_{12}^m}{S_{mL}}\right)^2 \leq 1 \text{ while } \sigma_{22}^m \geq 0 \tag{8}$$

Since the analysis concludes micro-scale modeling and the failure criteria consider the macro failure phenomenon, the proposed framework is named Trans-scale method. In conventional FE analysis or Classical Lamination Theory (CLT), strain response in each ply can be obtained and then the micro-stresses in the fiber and matrix phases are computed using the aforementioned RVE. The microscopic stress fields are obtained by Eq.9

$$\{\sigma\}^m = H^e\varepsilon + \Delta T H^T \tag{9}$$

where H^e is the stress field on the fiber-matrix scale obtained by the first 6 cases listed in Tab.1, ε the macroscopic strain in the ply, ΔT is the change of the temperature, H^T is the stress field at the fiber-matrix scale obtained by the last case listed in Tab.1. In order to obtain the microscopic stress field $\{\sigma\}^m$, the stress tensor H^e, at the Gaussian integration points in every element of the RVE are substituted into Eq.9. Then the failure index can be calculated by $\{\sigma\}^m$.

3 RESULTS AND DISCUSSION

To validate the micro-mechanical result obtained by RVE, analysis for laminates with $\left[0/90\right]_s$ and $\left[0/45/90\right]_s$ layer have been conducted. The results are compared with the results from literature [10] provided in Table 2, where, σ_1, σ_2 are the maximum and minimum principal stresses obtained by the RVE. $\sigma_{\max}$, $\sigma_{\min}$ are the corresponding results in the literature. The maximum error of $\sigma_{\max}$ in the fiber and matrix are 6.5% and 10.8%. The calculated results indicate that the micro-stress fields obtained through the RVE are of good agreement with previous studies.

Here, it should be note that, although the micro stresses are not considered in the modeling process in traditional macro analysis, it does not mean that the macro analysis is not accurate. The effect of the micro stresses is actually included by the measure matertial strength used in the macro analysis. Since then, the micro failure prediction results are compared with the LaRC03 prediction in Fig.3. The material properties used is listed in Tab.3. The micro prediction gives a good capture of the shear strength enhancement effect under transverse compression and in-plane shear.

Laminate		A		B		
Angle		0	90	0	45	90
Fiber (MPa)	σ_1	48.1	52.1	44.6	40.8	49.3
	$\sigma_{\max}$	51.2	55.2	47.7	43.3	52.2
	Error	6.1%	5.6%	6.5%	5.8%	5.6%
	σ_2	-78.3	-190.7	-126.1	-19.7	-261.3
	$\sigma_{\min}$	-91.0	-204	-147	-23.4	-283
	Error	14.0%	6.5%	14.2%	15.8%	7.7%
Matrix (MPa)	σ_1	58.8	62.4	61.1	55.6	63.4
	$\sigma_{\max}$	63.4	66.6	68.4	61.3	71.1
	Error	7.3%	6.3%	10.7%	9.3%	10.8%
	σ_2	-24.8	-26.1	-23.6	-22.2	-25.0
	$\sigma_{\min}$	-21.8	-22.9	-21.4	-19.7	-22.7
	Error	-13.8%	14.0%	-10.3%	-12.7%	-10.1%

Tab.2 The prediction results VS previous studies [10]

Fiber	AS4	Matrix	3501-6 epoxy
Longitudinal modulus E_{f1} (Gpa)	225	Elastic modulus E_m (Gpa)	4.2
Transverse modulus E_{f2} (Gpa)	15	Shear modulus G_m (Gpa)	1.567
In-plane shear modulus G_{f12} (Gpa)	15	Poisson's radio ν_m	0.34
Transverse shear modulus G_{f23} (Gpa)	7	tensile strength Y_{Tm} (Mpa)	69
Poisson's radio ν_{f12}	0.2	compressive strength Y_{Cm} (Mpa)	250

Tab.3 Material properties used in the prediction [3]

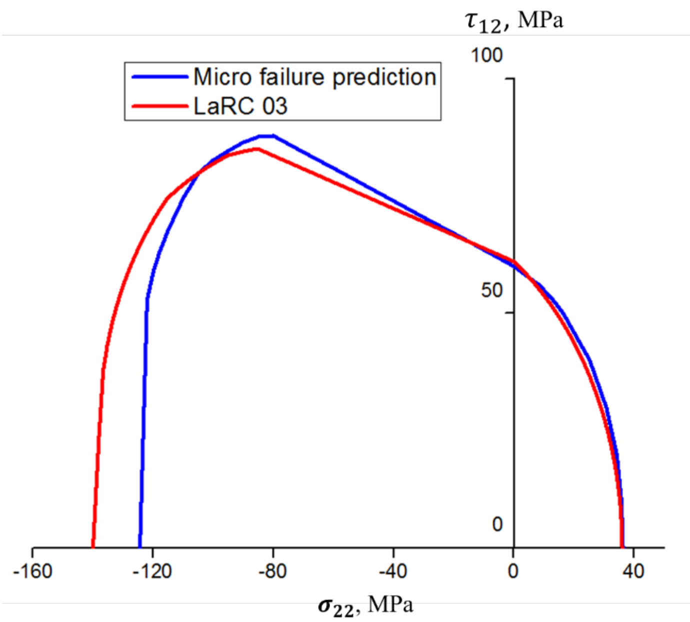

Fig.3 the predicted failure envelopes of micro analysis and LaRC03

However, when the temperature decreases, the matrix is under transverse tension stress due to the different thermal expansion coefficient between fiber and matrix. The micro analysis has confirmed this effect and can obtain the matrix stress fields. Therefore, when a relatively small transverse compression stress applied, the transverse stress in the matrix remains positive and the shear-compression failure mode should be dominated by the tension failure. The macro analysis can not consider this effect. It is to say, there is a particular stress range that may lead to misuse the tension and compression failure index for macro analysis. This is especially important for those criteria that have different formula for different stress state. The boundary of this effect can be detemined by micro-stress analysis, which will be a significant supplement for the failure prediction. This effect will be investigated intensively in the future work.

4 CONCLUSIONS

The trans-scale framework, which consists of the micro-scale modeling and the matrix failure criteria, is proposed in the present work. The method guarantees accurate simulations of the stress fields and the material failure envelope compared with the results published in literature. The micro analysis is able to obtain the matrix stress fields, which can be used to evaluate the boundary of the applicable range of the tension and compression failure index. The trans-scale framework proposed in this paper is believed to be favorable for the matrix faiure prediction under cryogenic temperature applications.

ACKNOWLEDGEMENTS

This research was supported by the National Basic Research Program of China (No. 2014CB046506, No. 2014CB049000), the National Natural Science Foundation of China (No.11372058), and the Fundamental Research Funds for the Central Universities (No. DUT13LK28).

Particularly, the author wants to thank Yan Zhou and Jie Cong from Dalian University of Technology for their helpful comments and suggestions.

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