Cohesive modelling of thin elasto-plastic pressure-dependent adhesive joints

N. Bordignon, A. Piccolroaz[1]

[1]Department of Civil, Environmental and Mechanical Engineering, University of Trento, Italy

Abstract

An adhesive joint between dissimilar elastic media is modelled as a thin layer made of an elasto-plastic pressure-sensitive material, which, in turn, is reduced to an imperfect zero-thickness interface equipped with non-linear transmission conditions. The transmission conditions are implemented in a cohesive model in the form of traction-separation laws. A comparison between cohesive elements and continuum elements is performed to validate the proposed approach.

Keywords: Adhesive joint; Non-linear transmission condition; Imperfect interface; Cohesive element

1 Introduction

The modelling of adhesive joints in composite materials and structures typically involves separate parts glued together by a very thin adhesive layer. In these situations, the intermediate layer may be considered to be of zero thickness [Mishuris and Öchsner, 2007] and consequently modelled through the use of cohesive elements [Zhang et al., 2017]. However, the constitutive modelling of the cohesive elements requires the definition of traction-separation laws accounting for the non-linear material behaviour of the glue material, which usually is a polymer and may exhibit large plastic deformations before failure [Wang and Chalkley, 2000]. Recently, Sonato et al. [2015] have obtained accurate transmission conditions for thin elasto-plastic pressure dependent adhesive layers. In the present paper, these transmission conditions are implemented into a cohesive element and the cohesive model is validated through a comparison with the continuum approach.

2 Formulation of the problem and the non-linear transmission conditions

Consider a structure composed of two dissimilar elastic materials joined together by a thin adhesive layer, see Fig. 1. The thickness of the layer is small in comparison with the characteristic size of the body. The adhesive material is assumed to be soft in comparison with the two adherents and may exhibit a very general non-linear constitutive behaviour, including compressible plastic deformations, the only assumption being that the material is isotropic.

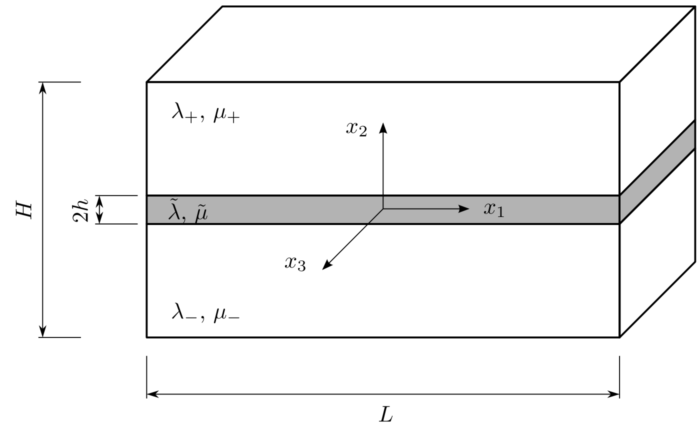

Figure 1: Bimaterial structure with thin soft adhesive joint.

Employing an asymptotic procedure, the adhesive layer is replaced by an imperfect interface of zero-thickness equipped with non-linear transmission conditions, see Sonato et al. [2015] for details. The transmission conditions, in plane strain, are given by the continuity of tractions across the interface

$$[\![\sigma_{12}]\!] = 0, \qquad [\![\sigma_{22}]\!] = 0, \tag{1}$$

together with a non-linear relationship between tractions and displacement jumps

$$\begin{pmatrix} \sigma_{12} \\ \sigma_{22} \end{pmatrix} = \frac{1}{2h} \begin{pmatrix} \tilde{\mu} & 0 \\ 0 & \tilde{\lambda} + 2\tilde{\mu} \end{pmatrix} \begin{pmatrix} [\![u_1]\!] \\ [\![u_2]\!] \end{pmatrix}, \tag{2}$$

Here, the notation $[\![f]\!] = f(x_2 = h) - f(x_2 = -h)$ stands for the jump of a function f across the imperfect interface and $\tilde{\lambda}$ and $\tilde{\mu}$ are generalized Lamé parameters given by

$$\tilde{\lambda}(\phi_1, \phi_2) = \frac{3\nu + (\phi_2 - \phi_1)E}{3(1 + \nu + \phi_2 E)(1 - 2\nu + \phi_1 E)} E, \quad \tilde{\mu}(\phi_2) = \frac{E}{2(1 + \nu + \phi_2 E)}, \tag{3}$$

where E is the Young modulus and ν the Poisson's ratio of the adhesive material. The two parameters ϕ_1 and ϕ_2 describe the deformation of a generic elasto-plastic pressure-sensitive material and may be defined when the elasto-plastic model for the adhesive material has been specified.

We assume a material obeying the Drucker-Prager yield criterion, given in terms of stress invariants $J_1^\sigma = \operatorname{tr} \boldsymbol{\sigma}$, $J_2^s = \frac{1}{2} \operatorname{tr} \boldsymbol{s}^2$, $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{\operatorname{tr} \boldsymbol{\sigma}}{3} \boldsymbol{I}$, as

$$F(J_1^\sigma, J_2^s) = \alpha J_1^\sigma + \sqrt{J_2^S} - k_s = 0, \tag{4}$$

where $\alpha > 0$ is called pressure-sensitivity index and assumed to be a constant while k_s is a hardening parameter, which is a positive function of the equivalent plastic strain, $J_2^{e^p} = \frac{1}{2} \operatorname{tr}(\boldsymbol{e}^p)^2$, $\boldsymbol{e}^p = \boldsymbol{\varepsilon}^p - \frac{\operatorname{tr} \boldsymbol{\varepsilon}^p}{3} \boldsymbol{I}$. In case of linear hardening, we have

$$k_s \left(\sqrt{J_2^{e^p}} \right) = \alpha J_1^\sigma + \sqrt{J_2^s} = \left(\alpha + \frac{1}{\sqrt{3}} \right) \left(\omega \sqrt{J_2^{e^p}} + \sigma_s \right), \tag{5}$$

where σ_s is the uniaxial yield stress and

$$\omega = \frac{\sqrt{3}E^p E}{(1+\nu^p)E - \nu E^p} = \frac{2E^p(\sqrt{3}+3\alpha)}{3}, \tag{6}$$

in which E^p is the plastic modulus and ν^p the plastic Poisson's ratio. Therefore, for Drucker-Prager plasticity, the parameters ϕ_1 and ϕ_2 become

$$\phi_1(J_1^\varepsilon, J_2^e) = \frac{6\alpha\left\{3\alpha(1+\nu)EJ_1^\varepsilon + (1-2\nu)\left[3E\sqrt{J_2^e} - \left(3\alpha+\sqrt{3}\right)(1+\nu)\sigma_s\right]\right\}}{E\left[3E(J_1^\varepsilon - 6\alpha\sqrt{J_2^e}) + \left(3\alpha+\sqrt{3}\right)(1+\nu)(\omega J_1^\varepsilon + 6\alpha\sigma_s)\right]}. \tag{7}$$

$$\phi_2(J_1^\varepsilon, J_2^e) = \frac{3\alpha(1+\nu)EJ_1^\varepsilon + (1-2\nu)[3E\sqrt{J_2^e} - (3\alpha+\sqrt{3})(1+\nu)\sigma_s]}{E\{18\alpha^2E\sqrt{J_2^e} - 3\alpha[EJ_1^\varepsilon - (1-2\nu)(w\sqrt{J_2^e}+\sigma_s)] + \sqrt{3}(1-2\nu)(w\sqrt{J_2^e}+\sigma_s)\}}. \tag{8}$$

Finally, in plane strain conditions, the strain tensor invariants are given by

$$J_1^\varepsilon = \frac{1}{2h}[\![u_2]\!], \qquad J_2^e = \frac{1}{12h^2}[\![u_2]\!]^2 + \frac{1}{16h^2}[\![u_1]\!]^2. \tag{9}$$

Note that the transmission conditions derived for the Drucker-Prager plasticity reduces to the Von Mises plasticity when $\alpha = 0$.

3 Cohesive element and implementation of the transmission conditions

The FE modelling of the adhesive layer is obtained through the use of cohesive elements. In the Abaqus element library, there is a 4-node two-dimensional cohesive element, denoted COH2D4, with two active degrees of freedom, x and y, as shown in Fig. 2.

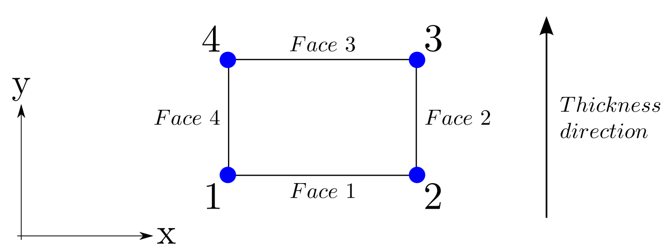

Figure 2: Geometry of the cohesive element COH2D4 implemented in Abaqus and the direction to define the element thickness.

The constitutive thickness of the cohesive element is assumed equal to one. This choice ensures that nominal strains are equal to the relative separation displacements. The nature of the mechanical constitutive response is based on a traction-separation description of the interface. In two-dimensional problems the traction-separation model assumes two components of separation, one normal to the interface, ε_{22}, and the other parallel to it, ε_{12}, and the corresponding stress components, the direct through-thickness stress σ_{22} and the transverse shear stress σ_{12}, are assumed to be active at a material point.

The FE program Abaqus allows to associate these elements with a linear elastic behaviour that follows the law:

$$\begin{pmatrix} \sigma_{22} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} K_{nn} & K_{ns} \\ K_{ns} & K_{ss} \end{pmatrix} \begin{pmatrix} \varepsilon_{22} \\ \varepsilon_{12} \end{pmatrix}, \tag{10}$$

where the stiffness matrix links the stress vector with the strain vector through a linear relation. This response is called traction-separation law. Using an external user material subroutine (UMAT), it is possible to implement a specialised traction-separation law that follows the non-linear transmission conditions described in Sec. 2. The new relation is given by

$$\begin{pmatrix} \sigma_{22} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} K_{nn}(\phi_1, \phi_2) & 0 \\ 0 & K_{ss}(\phi_1, \phi_2) \end{pmatrix} \begin{pmatrix} \varepsilon_{22} \\ \varepsilon_{12} \end{pmatrix}, \tag{11}$$

where the stiffness coefficients K_{nn} and K_{ss} are functions of ϕ_1 and ϕ_2 and are given by

$$K_{nn}(\phi_1, \phi_2) = \frac{1}{2h}\Big[\tilde{\lambda}(\phi_1, \phi_2) + 2\tilde{\mu}(\phi_1, \phi_2)\Big], \quad K_{ss}(\phi_1, \phi_2) = \frac{1}{2h}\tilde{\mu}(\phi_1, \phi_2). \tag{12}$$

4 Numerical simulations

Two-dimensional plane strain finite element simulations have been performed to validate the proposed cohesive model relate to the transmission conditions described in the Sec. 3. This numerical approach based on cohesive elements is different from the continuum approach used in Sonato et al. [2015]. In the following we present and compare results obtained with both approaches. In other words we perform two FE analysis for each loading condition: in the first analysis we make use of continuum elements to describe the adhesive layer of finite thickness, whereas in the second analysis we model the adhesive layer as an imperfect interface through cohesive elements.

4.1 Description of the numerical models

A structural joint is considered, consisting of two parts connected through an adhesive layer. The geometry and properties of the joint are shown in Fig. 3: two elastic blocks ($E = 72500$ MPa and $\nu = 0.34$), having geometrical dimensions $L = 10$ mm, $H = 1$ mm are glued together by an adhesive layer having thickness $2h = 0.01$ mm. The two blocks are modelled with 4-node bilinear elements with reduced integration and hourglass control (CPE4R).

The adhesive material is assumed to obey the Von Mises ($\alpha = 0$) or Drucker-Prager ($\alpha = 0.1504$) plasticity, with the following material properties: $E = 813$ MPa, $\nu = 0.3$, $\sigma_s = 50$ MPa, $E^p = 81.3$ MPa, $\nu^p = 0.22$ (for Von Mises), $\nu^p = 0.53$ (for Drucker-Prager). In the continuum approach, the adhesive layer is modelled with 4-node bilinear elements with reduced integration and hourglass control (CPE4R), and a strong mesh refinement is used in order to obtain accurate results. In particular, 40 elements have been used along the thickness direction, giving a total of 757599 elements for the discretization of the whole structure. In the cohesive approach, the adhesive layer is modelled with 4-node two-dimensional cohesive elements (COH2D4) with traction-separation law derived by the transmission conditions described in Sec. 2 and only one element is used along the thickness direction, giving a total of 34900 elements for the whole structure.

Two load conditions are considered: a tensile loading, Fig. 3a, and a combined loading consisting of a superposition of tensile and shear loading, Fig. 3b.

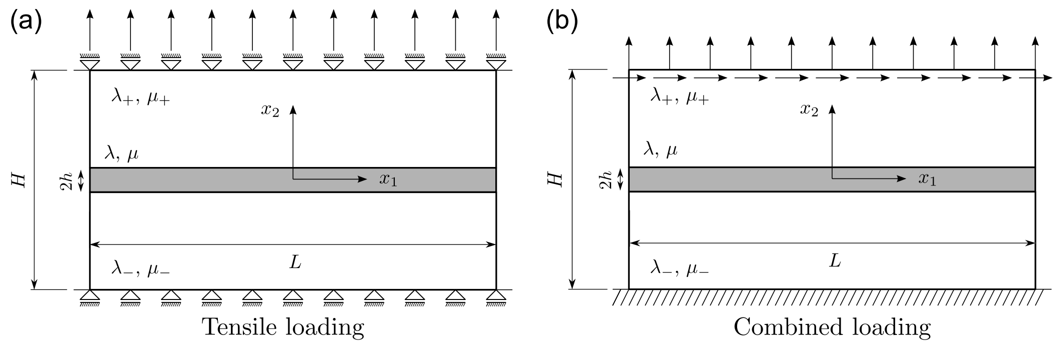

Figure 3: Load conditions applied to the structural joint: (a) tensile loading, (b) combined loading.

4.2 Numerical results

Results in terms of tractions along the interface for the two loading conditions sketched in Fig. 3 are reported in the following. In particular, Fig. 4 shows the normal traction σ_{22} along the interface produced by a tensile loading for an adhesive material obeying the Von Mises plasticity, part (a) of the figure, and the Drucker-Prager plasticity, part (b) of the figure. In both cases, the graph shows the results obtained with the continuum approach (green line) and with the cohesive approach (red line with markers). Fig. 5 shows the normal traction σ_{22} and the shear traction σ_{12} along the interface produced by a combined loading for an adhesive material obeying the Von Mises plasticity, part (a) and (b) of the figure, and the Drucker-Prager plasticity, part (c) and (d) of the figure. In all cases, the graph shows the results obtained with the continuum approach (green line) and with the cohesive approach (red line with markers).

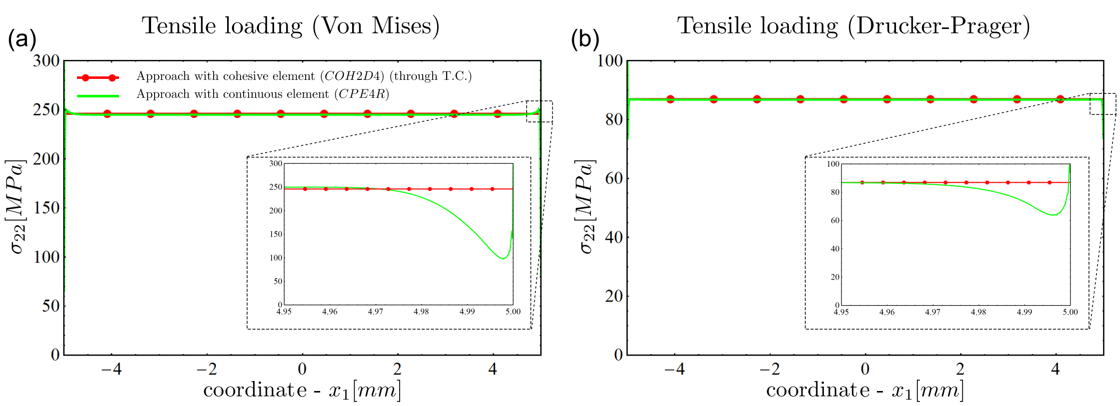

Figure 4: Distribution of the normal traction σ_{22} along the imperfect interface for tensile loading: (a) adhesive material obeying the Von Mises yield criterion, (b) adhesive material obeying the Ducker-Prager yield criterion. The inset is a magnification of the part of the graph in the vicinity of the right edge, showing the edge effects.

It is possible to note that the accuracy of the cohesive model (cohesive elements equipped with the non-linear transmission conditions) is very high and the results coincide with the continuum approach along the whole interface, except for a small edge zone, where edge effects take place. In

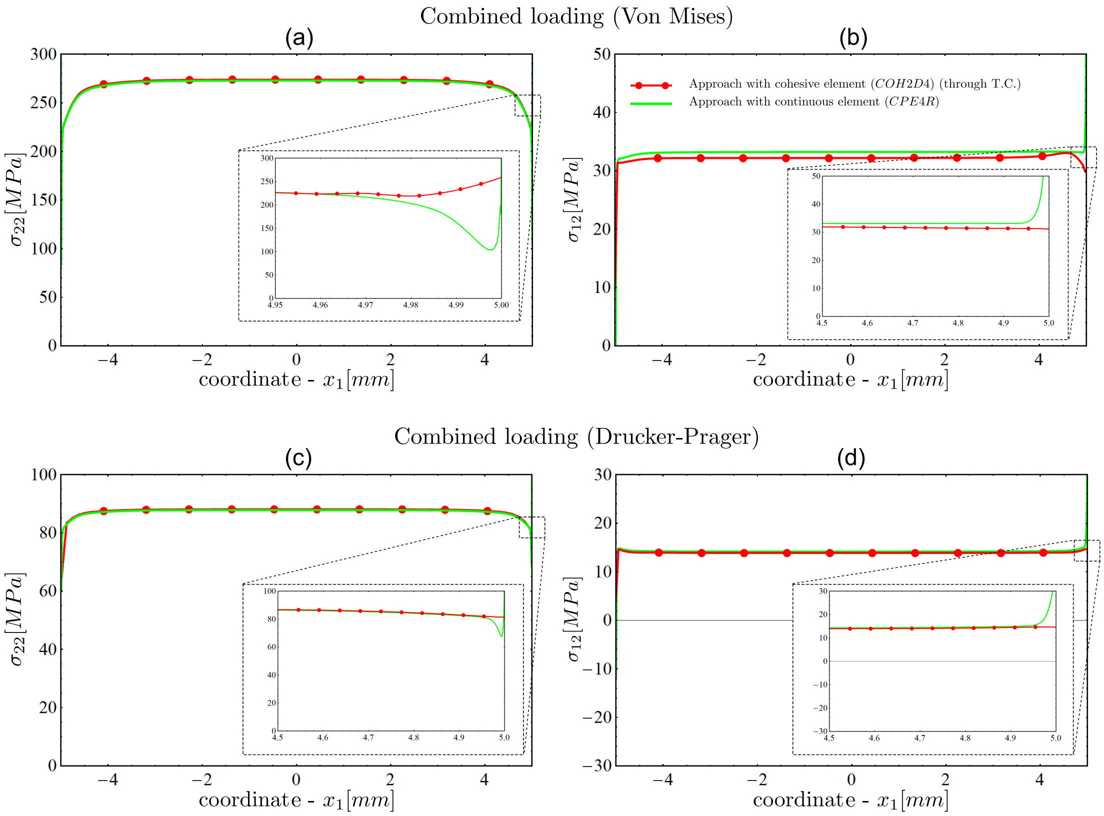

Figure 5: Distribution of the normal traction σ_{22} and shear traction σ_{12} along the imperfect interface for combined loading: (a) and (b) adhesive material obeying the Von Mises yield criterion, (c) and (d) adhesive material obeying the Ducker-Prager yield criterion. The inset is a magnification of the part of the graph in the vicinity of the right edge, showing the edge effects.

the continuum approach, there is an intensification of the interface tractions at the edge, due to the mismatch of material properties. This stress intensification is strongly reduced in the cohesive model.

5 Conclusions

A thin adhesive layer made of a soft elasto-plastic pressure dependent material can be effectively modelled using a cohesive model in which the traction-separation law are given by the transmission condition obtained in Sonato et al. [2015]. The advantage of the cohesive model is in the computational time which is greatly reduced with respect to that of the continuum approach, see Fig. 6.

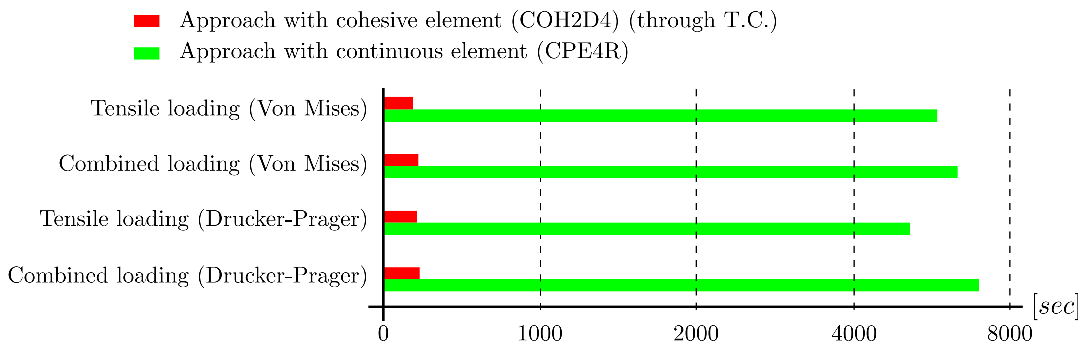

Figure 6: The computational times for the analyses with continuous elements (CPE4R) and with cohesive elements (COH2D4).

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