

SUBSONIC FLUTTER EMULATION OF COMPOSITE LAMINATE USING A FEW CONCENTRATED FORCES

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ABSTRACT

The concept of ground flutter simulation test is introduced to emulate the flutter of a composite laminate plates using a few concentrated forces. The concentrated forces are considered to be equivalent to aerodynamic forces and the equivalence is carried out using surface spline method and principle of virtual work. The present method differs to nominal flutter analysis in scheming generalized forces with invariant structural properties. Modal properties of composite laminate plates are obtained using a four-node quadrilateral finite element method and the aerodynamic forces are evaluated based on doublet-hybrid method. The flutter bounds of composite cantilever plate in subsonic flow is examined numerically using the V-g method. The location of concentrated forces and control points are determined by minimizing the difference between emulated modes and nominal modes. The numerical results include the influence of fiber orientation, lamina stacking sequences, aspect ratio, and Mach number on the flutter characteristics of emulated flutter model. The results are compared with nominal values and relatively good agreement of flutter velocities and flutter frequencies are obtained.

1 INTRODUCTION

Flutter is principle phenomenon of interest under dynamic aeroelasticity characterized by the mutual interaction of inertial, elastic, and aerodynamic forces. The essence of flutter prediction is very important to prevent catastrophic structural failure or undesirable limit cycle oscillation. As the modern aircraft are racing towards high speed and light weight, the selection of structure material to prevent aeroelastic instabilities is crucial. In general, most of the material used in aerospace industry is either conventional isotropic or composite material. Currently, the application of composite material is drastically increasing to reduce weight and to improve structure performance. This is mainly due to its superior characteristics, namely, high strength to stiffness and weight ratio, controllable, and versatility.

There exists a great quality of research works that discuss the benefits of using the composite material to enhance the flutter boundaries, investigated by means of analytical, numerical [1-3], and experimental [3] approaches. However, the analytical and numerical methods require mathematical modeling based on certain assumption. And, the modeling may be inadequate to represent the nonlinearities of the real structure leading to the error. Also, the experimental testing which is usually carried out using wind tunnel is complex and expensive for general research purpose. And, the testing requires scaling process of real structure that may induce discrepancy in the structural characteristics such as modal damping. Hence, the concept of ground flutter simulation test (GFST) has been studied as an alternative approach to overcome the deficiencies of traditional flutter prediction method. The general principle of the GFST is to apply few concentrated forces equivalent to aerodynamic forces to obtain close coupling between structure response and the aerodynamic forces in a real time. Thus, by measuring the response, flutter is observed. GFST was first proposed [4] in the early 1960s but the test outcomes were unsatisfactory mainly due to the limitation of technology. In 2011, ZONA Technology [5], conducted a comprehensive study of GFST followed by researchers from Beihang University [6] in 2016 but with a different approach to optimizing sensor and actuator location. Similar work on

panel flutter emulation using few concentrated forces numerically was reported [7] as well. However, their works on GFST are limited to the isotropic material.

The present work attempts to carry out the GFST concept in emulating the flutter of composite laminate plates. The emulated flutter model (EFM) is discretized using finite element method (FEM). The surface spline method and a principle of virtual work are introduced to obtain concentrated forces equivalent to distributed aerodynamic loading. Location of control points and concentrated forces points are optimized using flutter related modes. And, finally V - g method is used to obtain flutter velocity and flutter frequency.

2 MATHEMATICAL FORMULATION

The aeroelastic stability equation of motion of a composite laminate plates in a modal coordinate can be expressed as

$$\bar{M}\ddot{q} + \bar{C}\dot{q} + \bar{K}q = F_e \quad (1)$$

where, \bar{M} , \bar{C} , and \bar{K} are modal mass, damping, and stiffness matrices, and F_e is generalized aerodynamic forces. The formulation of aerodynamic force is based on doublet-hybrid method [8]. The method generates influence coefficient (AIC) matrix that relates structural deformation to aerodynamic forces at aerodynamic boxes. In contrast, the dynamics of EFM using a few concentrated forces and invariant structure properties can be expressed as

$$\bar{M}\ddot{q} + \bar{C}\dot{q} + \bar{K}q = F_c \quad (2)$$

where, F_c is generalized concentrated forces.

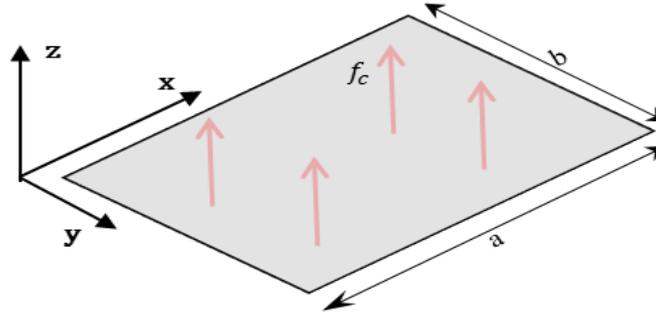


Figure 1: Composite laminate plate subjected to concentrated loading.

In EFM, the structure deflection is considered at a few control points (CP). Consequently, the deflection at aerodynamic boxes (w) is interpolated from control points deflection using interpolation matrix (G_w) from surface spline method [9]. Hence, the relationship between the forces acting on the aerodynamic boxes (f_d) and the deflection at control points (u_s) can be written as

$$f_d = q_\infty [AIC(k, Mach)] G_s u_s, \quad w = G_w u_s, \quad \Phi_w = G_w \Phi_s \quad (3)$$

Similarly, the deflection at aerodynamic boxes center of pressure (d), interpolated from concentrated forces point deflection (u_c) using interpolation matrix (G_d) is

$$d = G_d u_c, \quad \Phi_d = G_d \Phi_c \quad (4)$$

Based on the principle of virtual work, the two force system should do same virtual work in their respective deflection modes. Thus, the concentrated forces acting on the laminate plate can be obtained using above equation.

$$\begin{aligned} f_c &= G_d^T f_d \\ &= G_d^T q_\infty [AIC(k, Mach)] G_s u_s \end{aligned} \quad (5)$$

Hence, the generalized forces of emulated flutter model are obtained using mode superposition method ($u_s = \Phi_s q$) which can be written as

$$F_c = \Phi_c^T f_c = \Phi_c^T G_d^T q_\infty [AIC(k, Mach)] G_s \Phi_s q \quad (6)$$

Finally, the flutter speed and flutter frequency of EFM are obtained by solving Eq. (2) using the *V-g* method.

3 LOCATION OPTIMIZATION

The location of control points and concentrated forces points (CFP) are significant parameters to assure the emulated flutter boundaries closer to nominal flutter boundaries. The closer flutter boundaries can be assured when the Eqs. (1) and (2) have more similar generalized forces by having similar modes. Hence, the location CPs and CFPs can be optimized by minimizing the difference between the emulated modes and nominal modes. The following Frobenius norm as an objective function is chosen to optimize the CPs location.

$$J_w = \|(\bar{\Phi}_w - \Phi_w)\eta\|_F \quad (7)$$

Similarly, the objective function for optimizing CFPs location is

$$J_d = \|(\bar{\Phi}_d - \Phi_d)\eta\|_F \quad (8)$$

where, $\bar{\Phi}_w$, and $\bar{\Phi}_d$ are modal matrices from nominal flutter. It is realized that the contribution of higher modes in flutter boundaries prediction is relatively small. Thus, the weight coefficient matrix (η) is given to increase the effectiveness of minimizing the objective functions.

For experiment purpose, it is desirable to have fewer CPs and CFPs to reduce control problems and cost. Since the CPs deflection are measured using a sensor of small volume in counterpart to CFP deflection using shakers. A larger number of CPs than CFPs can be used without compromising the results.

4 RESULTS AND DISCUSSION

Numerical analysis is performed for both isotropic and composite laminate plates to demonstrate the accuracy of the present method. FEM models with distributed loading are taken to obtain nominal (reference) flutter values. A fine mesh with an adequate grid of aerodynamic boxes is considered. For location optimization, the modes contributing more, typically first bending and first torsional modes are assigned unit weight coefficient while other higher modes are given lower weight values. Six CPs and three CFPs are used in emulated flutter calculation. Furthermore, the parametric studies are conducted for different laminate aspect ratio, fiber orientation, and Mach number to observed the consistency of emulated flutter values to nominal values.

4.1 ISOTROPIC

A clamped aluminum plate with a dimension of 0.5 x 0.4 x 0.005 m and properties: $E = 70\text{GPa}$, $\rho = 2700\text{kg/m}^3$, $\nu = 0.3$ is considered. The results for both nominal and emulated flutter are shown in Table 1. The flutter values agree quite well with those provided by GFST [6].

	Present method		GFST	
	Nominal	Emulated	Nominal	Emulated
Flutter speed (m/sec)	259.399	260.627	260.61	262.32
Flutter frequency (Hz)	31.340	30.461	31.96	32.22

Table 1: Nominal and emulated flutter boundaries of the isotropic plate.

4.2 COMPOSITE LAMINATE

At first, the nominal flutter response of graphite/epoxy composite laminate is obtained and compared with the available references as shown in Table 2. A good agreement of flutter velocity and flutter frequency of two different lamina stacking sequences is observed.

	Nominal flutter			
	$[0_2/90]_s$		$[30_2/0]_s$	
	V_f (m/sec)	ω_f (Hz)	V_f (m/sec)	ω_f (Hz)
Present method	22.54	24.54	27.44	25.84
Computation [3]	21.8	25.6	24.9	26.4
Computation [2]	21.0	25.0	27.8	31.0
Experiment [2]	25.0	29.0	27.0	28.0

Table 2: Nominal flutter comparison of composite cantilever plate.

Engineering constant	Value
E_{11}	[GPa] 213.74
E_{22}	[GPa] 18.62
ν_{12}	0.28
G_{12}	[GPa] 5.17
Ply thickness (t_p)	[m] 0.3333×10^{-3}
ρ	[kg/m ³] 2051.88

Table 3: Boron/Epoxy engineering constants.

Next, the composite laminate composed of six boron/epoxy lamina is chosen to study the effect of fiber orientation on the emulated flutter behavior. The laminate has chord length 300 mm (AR=1.5) and its engineering constants are given in Table 3. The numerical analysis is carried out for laminates $[\pm\theta/0]_s$ where θ runs through 0–45 deg. and the results are plotted in Fig. 2. It is noted that the flutter velocity increase with increasing fiber orientation with a relatively close value between nominal and emulated flutter method. Additionally, for symmetric laminate, the flutter value tend to decrease when $\theta > 45^\circ$ for which the flutter results are not presented here. Similarly, Fig. 3 shows the flutter characteristics of the laminate $[\pm 30/0]_s$ obtained using the V-g method and its flutter value and natural frequency are listed in Table 4. The result shows the similar trend of flutter characteristics between nominal and emulated flutter analysis. The corresponding optimization results are shown in Fig. 4 where virtual CPs and CFPs are introduced at the clamped edge to improve the accuracy of emulated flutter analysis. The results suggest that three concentrated forces are adequate to simulate real flutter.

Natural frequency (Hz)				Nominal flutter		Emulated flutter	
1 st B	1 st T	2 nd B	2 nd T	V_f (m/sec)	ω_f (Hz)	V_f (m/sec)	ω_f (Hz)
11.52	38.23	68.38	107.67	83.288	24.859	84.789	24.976

Table 4: Natural frequencies and flutter boundaries of $[\pm 30/0]_s$ (AR = 1.5).

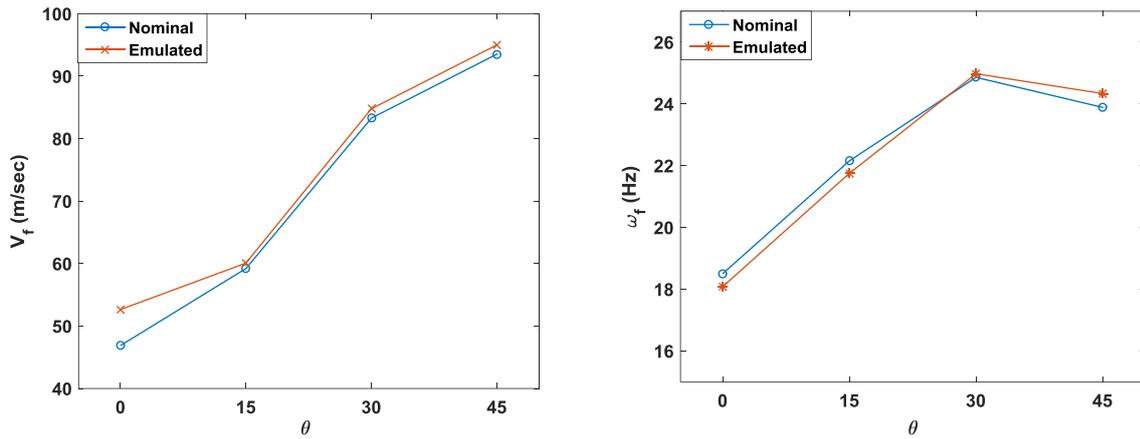


Figure 2: Effect of fiber orientation on emulated and nominal flutter boundaries for $[\pm\theta/0]_s$ (AR=1.5).

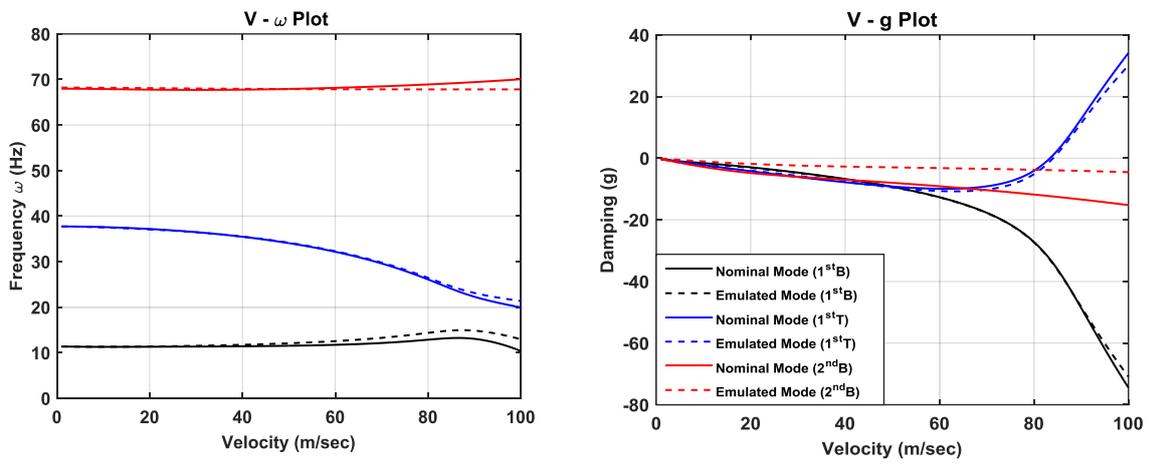


Figure 3: Comparison of flutter characteristic of laminate $[\pm30/0]_s$ (AR = 1.5).

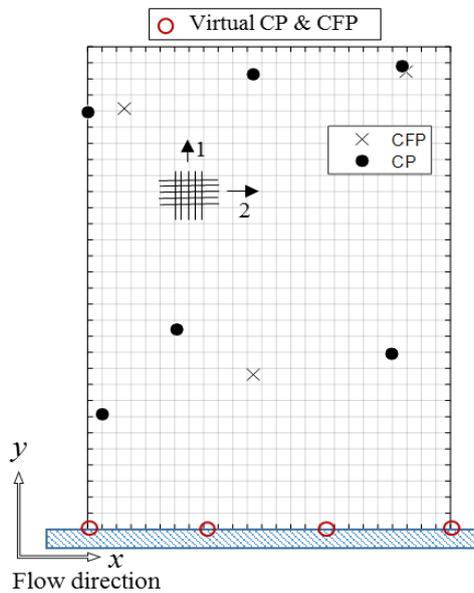


Figure 4: Optimization of CP and CFP $[\pm30/0]_s$ (AR=1.5).

A series of numerical analysis are conducted to investigate the effect of laminate aspect ratios and Mach numbers on both emulated and nominal flutter boundaries. The corresponding flutter velocities and flutter frequencies are presented in Figs. 5 and 6 for the fiber orientation from 0–45 deg., an aspect ratio from 1–4, and Mach number from 0.3–0.7. Typically, flutter occurs due to the coupling of the first bending and the first torsional modes and the frequency of these mode decrease as the aspect ratio increase. As a result, for a given fiber orientation, the flutter velocity and flutter frequency tends to decrease while gain in flutter bounds for increasing the fiber orientation as shown in Fig. 5. It is also observed that the emulated and nominal flutter velocities are closer for higher aspect ratio than lower aspect ratio. This is typically due to the farther location of CPs as well as CFPs in minimizing J_w and J_d effectively. Additionally, a similar flutter frequency is noticed for laminates of 45° fiber orientation and aspect ratio of 3.0 and 4.0. From the investigation, it is realized that the flutter is due to second bending rather than the first torsional mode for AR = 3.0. However, for AR = 4.0, the flutter is due to second bending when damping is added. Therefore, the emulated flutter results for these two case are obtained with additional CPs and CFPs. Furthermore, the effect of Mach numbers on flutter boundaries of the laminate $[\pm 30/0]_s$ is presented in Fig. 5. It is seen that change in flutter velocity and flutter frequency is insignificant for increasing Mach number.

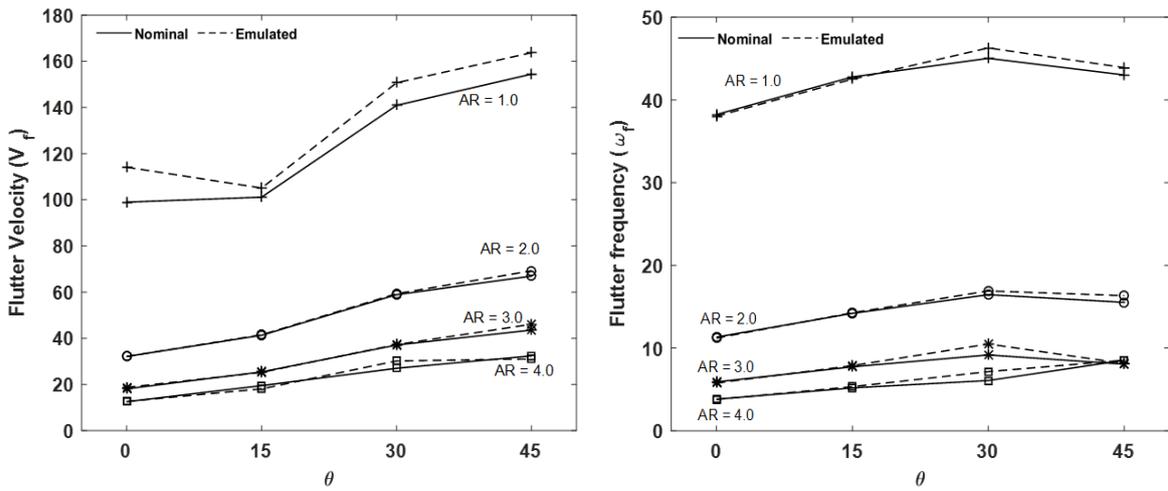


Figure 5: Effects of aspect ratio and fiber orientation on flutter boundaries of $[\pm\theta/0]_s$ laminates.

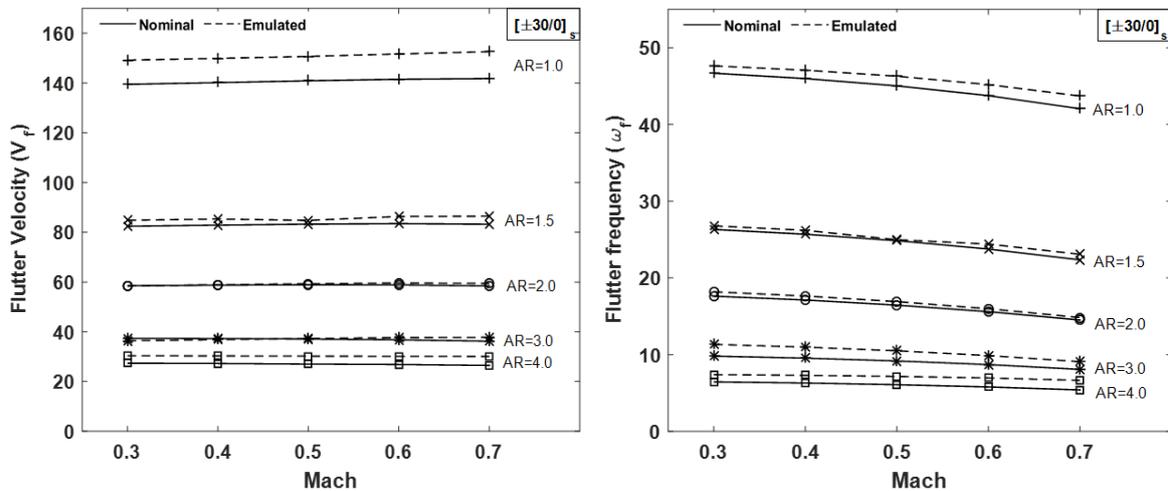


Figure 6: Effect of Mach number on flutter boundaries of $[\pm 30/0]_s$ laminates.

5 CONCLUSION

In this investigation, the concept of GFST is implemented to emulate the flutter of composite laminates numerically. The method requires the equivalence between concentrated forces and aerodynamic forces, are obtained using surface spline interpolation and principle of virtual work. The location of CPs and CFPs are based on the optimization scheme of minimizing the difference between the primary modes of nominal and emulated flutter model with additional virtual CPs and CFPs along the boundaries to produce accurate flutter results. The validity of the method is presented for an isotropic plate with a very good agreement in both emulated and nominal flutter. Further validation includes that of the symmetric composite laminates with the good agreement of nominal flutter velocity and frequency from other references.

Parametric studies are performed to investigate the effects of fiber orientation, aspect ratio, and Mach number on the flutter boundaries of composite laminates. For the given aspect ratio, increasing fiber orientation delays the flutter but the reverse effect on flutter is found for increasing aspect ratio for a constant fiber orientation. As for the Mach number effect, the change in the flutter boundaries is relatively negligible. All of the emulated flutter results are calculated using six CP and three CFP. However, these numbers can be increased to compensate the error. In all, the flutter velocity and frequency from EFM are relatively close with respect to nominal results.

It is pointed out that the present work is primarily a numerical approach to investigate the feasibility of flutter emulation of the composite laminate using few concentrated forces. Therefore, the results provided herein are significant in conducting the GFST of composite laminates.

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