DYNAMIC HOMOGENIZATION FOR PHONONIC METASOLIDS WITH CONSIDERATION OF WILLIS COUPLING

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1 INTRODUCTION

Metamaterials are artificial composites or structures varying in space at a scale that is much smaller than the operating wavelength [1]. They can exhibit properties which go beyond the realm provided by nature, such as negative effective material properties and negative refractive indices [2-5]. Numerous works have thus been devoted to the design and the development of acoustic and elastic metamaterials (also named metafluids or metasolids), with broad scientific and practical implications into harnessing the propagation of classical waves [6, 7]. The macroscopic properties of metamaterials are often obtained from effective medium theory. The determination of the frequency-dependent dynamic effective material constants, including the Willis coupling tensors relating strain to momentum and stress to velocity [8], has in this regard become a challenging topic.

2 PRESENT WORK

In this paper, we propose and discuss a method for the estimation of the dynamic effective properties of phononic metasolids that is based on the solution of definite boundary value problems. Effective constants, including the Willis coupling tensors, are estimated by using the volume integration of microscopic field variables in response to stimulations exerted on the boundary of a unit cell. Specifically, the method relies on the following three assumptions: (i) the Willis constitutive relation holds; (ii) this constitutive relation is for macroscopic quantities obtained as volume averages of the corresponding microscopic quantities over the unit cell; (iii) effective material constants are obtained as the response to elementary excitations applied on the boundary of the unit cell.

3 DISCUSSIONS

The proposed volume integration method has been discussed and compared analytically and numerically with the boundary integration method. The proposed estimation formulas for effective material constants have been proved to be consistent in the homogeneous test case and to have faster convergence with mesh size compared to the boundary integration method.

In the case of a phononic crystal presenting Bragg band gaps, effective constants remain single-valued well beyond the low frequency range corresponding to the long wavelength approximation. In the case of a phononic metasolid presenting locally-resonant band gaps, the range of existence of negative mass densities, defined as macroscopic force divided by macroscopic acceleration and volume of the unit cell, was not found to match the band gap ranges, as found for instance with the boundary integration method. The apparent discrepancy is readily solved by observing that the macroscopic force, \(-\omega^2\langle \rho u \rangle\), has indeed a sign opposite to that of the imposed boundary acceleration, \(-\omega^2 U^0_j\), as it should, but that the same property applies to the macroscopic acceleration, \(-\omega^2\langle u_j \rangle/V\). According to the proposed volume integration method, finally, the two Willis coupling tensors can be estimated but are not found to be each other’s transpose, because they are not obtained from the same boundary value problems.
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REFERENCES


