A SCALED BOUNDARY FINITE ELEMENT FORMULATION FOR COMPOSITE BEAMS OF ARBITRARY CROSS-SECTION

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ABSTRACT

For lightweight applications the so-called scaled boundary finite element method has been extended to the situation of composite beams.

1 INTRODUCTION

Beams and beam-like structures are widely used in mechanical and civil engineering. Due to lightweight reasons these beams are often made of thin-walled sections. When fibre-reinforced polymers are used the sections can be composite laminates where the laminate layup can be adjusted to given needs. For the analysis of such beams an effective and reliable computational method is required.

2 THE SCALED BOUNDARY FINITE ELEMENT METHOD

The scaled boundary finite element method (SBFEM) is such a method. It is a semi-analytical method where only the boundary is discretized and an analytical solution is used within the body. So it combines the benefits of the boundary element method and the finite element method without adopting the shortcomings [1], [2].

In the current work within the framework of first-order shear deformation theory beam sections of arbitrary layup are allowed. For the displacement field a separation ansatz of the kind $N(\eta)u(\xi)$ has been made where ξ is a coordinate in the longitudinal scaling direction and η is a coordinate running along the cross-section. N denotes a standard finite element shape function.

When this displacement representation is used the principle of minimal total potential leads to a set of differential equations for the unknowns u which can be solved through a corresponding eigenvalue problem. The respective free integration constants can be identified from the given boundary conditions. For details please consider [3].

The decisive advantage of the SBFEM in the end is that it requires much less elements than the finite element method and that it is numerically more efficient in many cases.

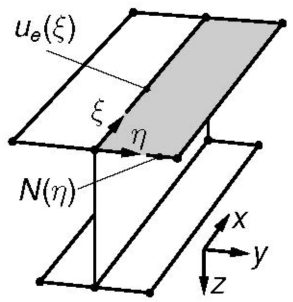

Figure 1: Thin-walled beam with scaling coordinates.

3 SBFEM IMPLEMENTATION

For the shape functions along the cross-section four different versions have been considered: a linear representation (quadratic in w), a 5th order variant, a linear mixed hybrid variant and a quadratic mixed hybrid variant. In all cases the respective differential equation system has been solved using matrix exponential functions [2]. The actual numerical implementation has been performed by the use of MATLAB. As test cases several cantilever beams with various cross-sections and various laminate layups have been considered. In figure 2 as an example the convergence error for the displacement w for a box-beam under concentrated single force is displayed, revealing a rapid convergence with an increasing number of degrees of freedom.

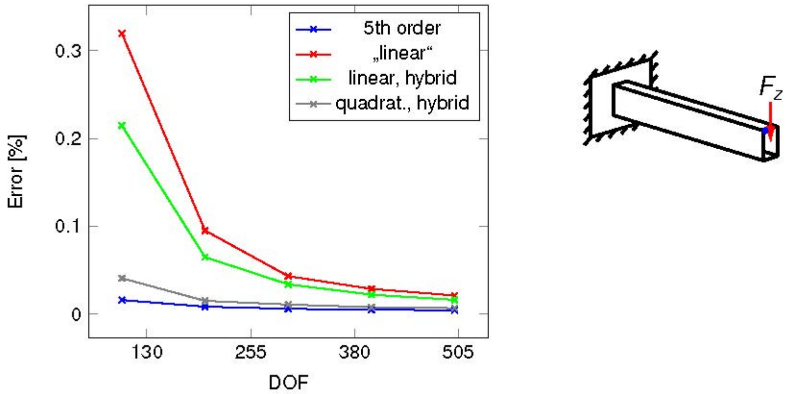

Figure 2: Displacement w for a box-beam (1mm, symmetric laminate).

Beyond the use of an exclusive scaled boundary finite element model also the implementation of SBFEM superelements for the embedding into larger finite element models has been performed and verified. This turns also out to lead to an enhanced numerical efficiency together with the advantages of the SBFEM to give good local information about the resultant mechanical field quantities.

4 CONCLUSIONS

It can be concluded that the implemented scaled boundary finite element method gives good results, even with rather few elements. The elements of higher order are better than the simpler linear elements. In case of slender beams the SBFEM clearly is faster than the finite element method. Eventually also an efficient generation of superelements is possible.

ACKNOWLEDGEMENTS

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