A NOVEL MULTISCALE METHOD FOR MECHANICAL PROPERTIES OF STATISTICALLY INHOMOGENEOUS MATERIALS

Zhiqiang Yang[1, *, †], Junzhi Cui[2], Yi Sun[1]
[1]*Department of Astronautic Science and Mechanics, Harbin Institute of Technology, Harbin, 150001 China*
[2]*LSEC, ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, 100190 China*

Keywords: statistically inhomogeneous materials; multiscale method; core-shell particle; effective property

ABSTRACT

This paper develops a novel multiscale method to predict mechanical performance of statistically inhomogeneous materials. As for these kinds of materials, the sophisticated microscopic information of inclusions, including their shape, size, orientation, spatial distribution, volume fraction and so on, leads to changes of the macroscopic mechanical properties. The multiscale formulae for solving the mechanical problems are proposed, and the associated prediction algorithm based on the two-scale model is brought forward in detail. Finally, some numerical results for the effective properties of the materials with varying probability distribution models are calculated by the algorithm proposed, and compared with the data by experimental results.

1 INTRODUCTION

This paper focuses on prediction of the mechanical properties which arises in statistically inhomogeneous core-shell materials. In particular, the treatments of these kinds of materials that have the complicated and varied microscopic configurations. Under these conditions, it is not possible to analyze all microscale heterogeneities of the inhomogeneous core-shell materials even using the advanced supercomputers with large mass storage capabilities. Therefore, to overcome this difficulty, homogenized methods and the associated multiscale methods are proposed to solve the problems. By introducing two length scales, i.e. macroscale and microscale, one can effectively analyze the inhomogeneous materials with sophisticated microstructures, which cannot only save efficiency but also ensure the calculation precision.

2 MULTISCALE FORMULATIONS

$u_{\varepsilon i}(x,\omega)$ $(i=1,2,3)$ can be expanded into a series in the following forms:

*Correspondence to: Zhiqiang Yang, Department of Astronautic Science and Mechanics, Harbin Institute of Technology, Harbin, 150001 China
†E-mail: yangzhiqiang@hit.edu.cn

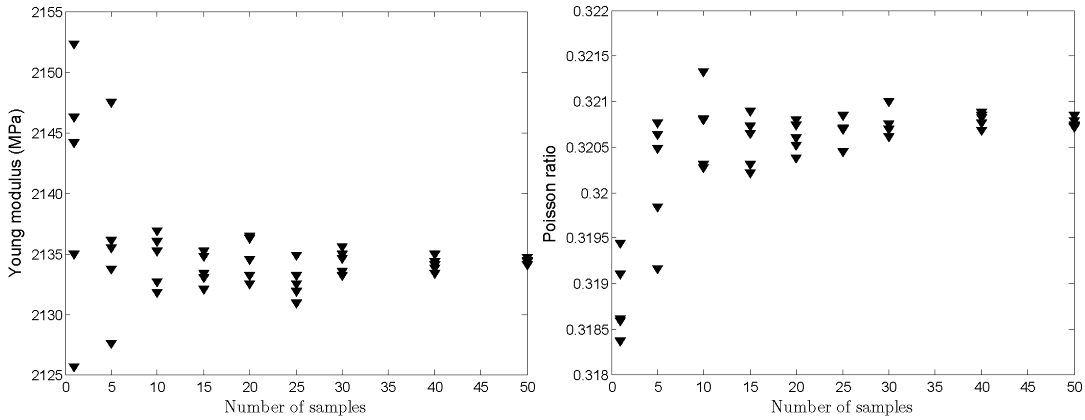
$$\mathbf{u}_\varepsilon(x,\omega)=\mathbf{u}_0+\varepsilon\mathbf{M}_{\alpha_1}(y,\omega_{x_0}^s)\frac{\partial\mathbf{u}_0}{\partial x_{\alpha_1}}+\varepsilon^2(\mathbf{M}_{\alpha_1\alpha_2}(x,y,\omega_{x_0}^s)\frac{\partial^2\mathbf{u}_0}{\partial x_{\alpha_1}\partial x_{\alpha_2}}$$

$$+\mathbf{B}_{\alpha_1}(x,y,\omega_{x_0}^s)\frac{\partial\mathbf{u}_0}{\partial x_{\alpha_1}})+O(\varepsilon^3),\ \ x\in\varepsilon Y_{x_0}^s\subset\Omega,\ y\in Y_{x_0}^s,\ \omega_{x_0}^s\in P(x_0)$$

where $\mathbf{u}_0=\mathbf{u}_0(x)$ only reflects the macroscopic behaviors, and is called the homogenized solution; $\mathbf{M}_{\alpha_1}(y,\omega_{x_0}^s)$, $\mathbf{M}_{\alpha_1\alpha_2}(x,y,\omega_{x_0}^s)$ and $\mathbf{B}_{\alpha_1}(x,y,\omega_{x_0}^s)$ $(\alpha_1,\alpha_2=1,2,3)$ $(s=1,2,3,...)$ contain microstructural information of the materials, and they are called the auxiliary functions defined on unit cell $Y_{x_0}^s$.

3 VALIDATION OF MULTISCALE METHOD

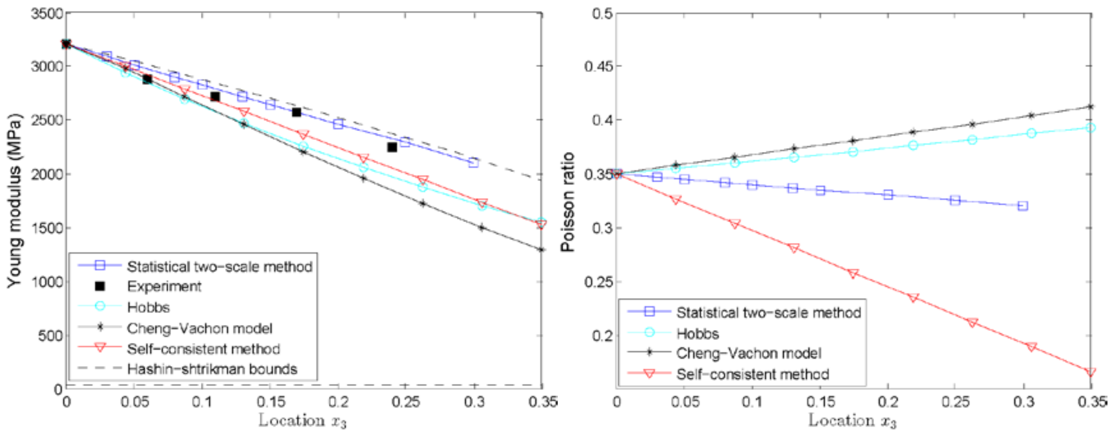
Figure 1 with the expected homogenized Young's modulus and Poisson's ratio in different numbers of samples illustrates the convergence of the expected homogenized parameters for volume fraction of 30% with uniform distribution of particles. Statistically, as shown in Figure 1, the different samples should have different results. However, accompanied by the increasing number of random samples with the same statistical characteristic, the mathematical expected value of the computational results should converge. As a results, for 30 or more samples a good prediction is obtained.



(a) (b)

Figure 1. (a) Effective Young's modulus and (b) effective Poisson's ratio of a core-shell materials for the different sample numbers.

Figure 2 displays the obtained values of the mechanical parameters from the statistical multiscale method as well as other computed values i.e. Hobbs [1], self-consistent method [2], Cheng-Vachon model [3] and Hashin-Shtrikman bounds [4]. The expected results based on the proposed statistical two-scale method agree reasonably well with the experimental results for uniform distribution of particles and fall into the Hashin-Shtrikman bounds [4]. Moreover, the numerical results based on the statistical two-scale method give better approximations of the empirical results at lower volume fraction, but the deviations of the results increase as the volume fraction of particles increasing.



(a) (b)

Figure 2. Comparison of the effective mechanical parameters obtained from experimental results [39] and different numerical models (a) Young's modulus; (b) Poisson's ratio.

4 CONSLUSION

This paper develops the multiscale method and associated numerical algorithms based on second-order two-scale asymptotic expressions for mechanical parameters prediction of statistically inhomogeneous core-shell materials. The two-scale asymptotic expansion formulas for the problems are derived, and the two-scale finite element algorithms are given in detail. Then, for the materials with random distribution of larger numbers of core-shell particles, the method can be used to predict the effective stiffness parameters and strength parameters.

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