MECHANICAL PROPERTIES OF 3D RE-ENTRANT AUXETIC CELLULAR STRUCTURES

Xin-Tao Wang, Bing Wang, Xiao-Wen Li, Li Ma*
* Center for Composite Materials, Harbin Institute of Technology, Harbin 150080, PR China
E-mail address: mali@hit.edu.cn (Li Ma), website: http://homepage.hit.edu.cn/mali

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ABSTRACT

In this work, an analytical model of a 3D re-entrant auxetic cellular structure has been established based on energy method. Analytical solutions for the modulus and Poisson's ratios of the cellular structure in all principal directions were deduced. The results show that when the struts are relative stubby, the overlapping of the struts as well as axial extension or compression should be taken into consideration for modelling.

1 INTRODUCTION

Auxetic was firstly coined by Evans to define materials with a negative Poisson's ratio for convenience [1]. It has been suggested that the negative Poisson's ratio character makes auxetic materials exhibit excellent mechanical properties such as increased shear modulus [2], superior indentation resistance [3], improved fracture toughness [4], higher energy absorption ability [5], porosity/permeability variation with strain [6], unique synclastic curvature [7]. Due to these excellent properties, auxetic materials have great potential for many applications fields [8]. As promising metamaterials, auxetic materials have attracted great interest.

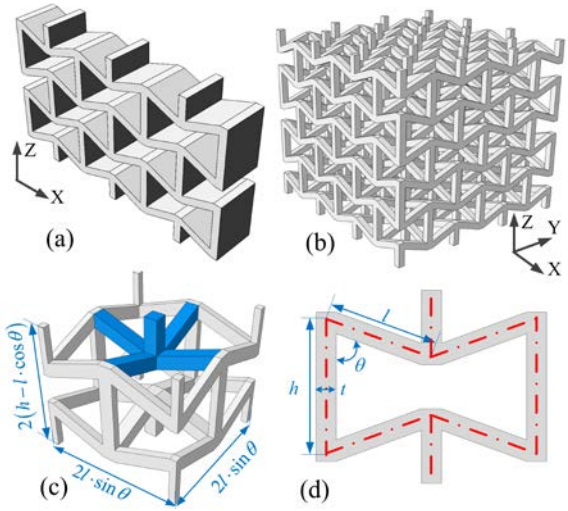

Figure 1 : (a) 2D re-entrant auxetic structure. (b) 3D re-entrant auxetic structure. (c) Unit cell of the 3D auxetic structure and its dimensions. (d) Geometry parameters.

The mechanical property of a 3D re-entrant auxetic cellular structure that exhibits auxetic behavior in all three principal directions is of concern in this paper, as shown in Figure 1(b). The 3D structure is extended from the 2D re-entrant hexagonal honeycomb structure [9], as shown in Figure 1(a). In this paper, the 3D re-entrant auxetic cellular structure was theoretical analyzed based on energy method. Analytical solutions for the modulus and Poisson's ratios of the cellular structure in all principal directions were deduced.

2 STRUCTURAL DESIGN AND SIMPLIFICATION

The unit cell of the structure is shown in Figure 1(c). The mechanical properties along X and Y axes are identical due to the geometry symmetry. For simplicity, all the struts were assumed to have the same square cross section. Then the geometry of structure can be described by four parameters: length of vertical strut (h), length of oblique strut (l), re-entrant angle between the vertical strut and the oblique strut (θ), and the side length of the strut cross section (t), as shown in Figure 1(d).

For the modeling, some simplifications were made. Firstly, the boundary effects need not to be considered. Secondly, deformation mechanisms of the structure consist of flexure, stretching and shearing. Thirdly, strains are small enough that gross changes in geometry do not occur. Fourthly, all of the joints in the structure are considered to be rigid. Because of the overlapping as shown in Figure 2, the effective lengths of both types of struts are shorter than the design values.

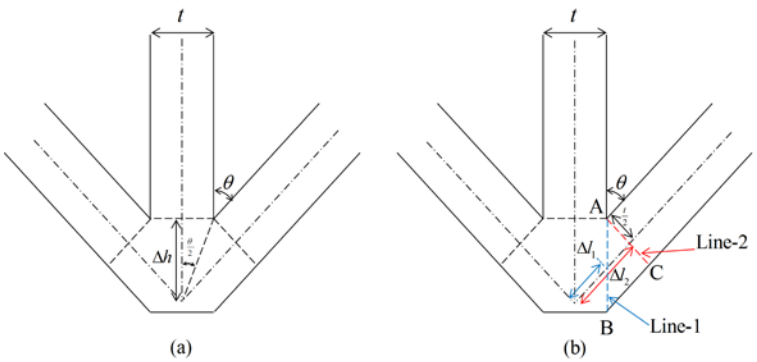

Figure 2: (a) Estimation of the length reduction of the vertical strut. (b) Estimation of the length reduction of the oblique strut.

As shown in Figure 2(a), the length reduction of vertical strut at one end can be estimated as

$$\Delta h = \frac{t/2}{\tan(\theta/2)} \tag{1}$$

As shown in Figure 2(b), the length reduction of oblique strut at one end can be estimated as

$$\Delta l = \varphi \Delta l_1 + (1-\varphi)\Delta l_2 \qquad 0 \le \varphi \le 1 \tag{2}$$

where

$$\Delta l_1 = \frac{t/2}{\sin\theta} \text{ and } \Delta l_2 = \frac{t/2}{\tan(\theta/2)} \tag{3}$$

and the coefficient φ is an experiential parameter. It was found that $\varphi = 0.6$ can make the analytical results agree well with the numerical results.

3 MODELING

3.1 Uniaxial compression in Z direction

Due to the symmetry of the 3D structure in X direction and in Y direction, the 2D re-entrant structure can be used to analyze the 3D structures under compression loading along Z direction. And the representative element $RQR'Q'$ was used for analysis, as shown in Figure 3(b).

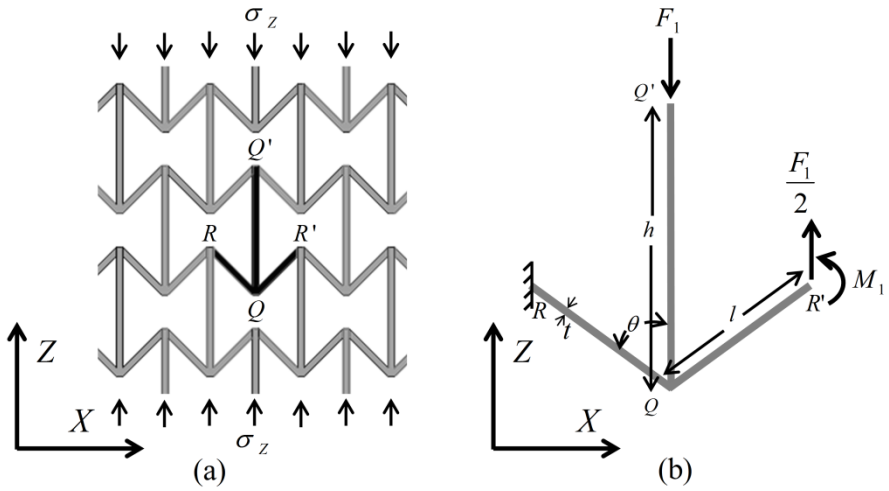

Figure 3: (a) The 2D re-entrant structure and the arrow element used to analyze the 3D structure. (b) Loading condition of the arrow element $RQR'Q'$.

Then, Poisson's ratio in Z direction under compressive stress can be obtained as

$$v_{ZX} = -\frac{\varepsilon_X}{\varepsilon_Z} = \frac{\dfrac{\Delta_X}{2l\sin\theta}}{\dfrac{\Delta_Z}{h - l\cos\theta}} = -\frac{h - l\cos\theta}{2l\sin\theta}\frac{\Delta_X}{\Delta_Z} \tag{4}$$

The effective modulus of the re-entrant cellular structure can be determined as

$$E_Z = \frac{\sigma_Z}{\varepsilon_Z} = \frac{\dfrac{4F_1}{(2l\sin\theta)^2}}{\dfrac{\Delta_Z}{h - l\cos\theta}} = \frac{F_1\left(h - l\cos\theta\right)}{\Delta_Z l^2 \sin^2\theta} \tag{5}$$

where

$$\Delta_X = \frac{F_1\left(l - 2\Delta l\right)^3 \sin\theta\cos\theta}{12E_s I} - \frac{F_1\left(l - 2\Delta l\right)\sin\theta\cos\theta}{E_s A} + \frac{6F_1\left(l - 2\Delta l\right)\sin\theta\cos\theta}{5G_s A} \tag{6}$$

and

$$\Delta_Z = \frac{F_1\left(l - 2\Delta l\right)^3 \sin^2\theta}{24E_s I} + \frac{F_1\left(l - 2\Delta l\right)\cos^2\theta}{2E_s A} + \frac{3F_1\left(l - 2\Delta l\right)\sin^2\theta}{5G_s A} + 2\frac{F_1\left(h - 2\Delta h\right)}{E_s A} \tag{7}$$

E_s and G_s are the Young's modulus and shear modulus of the solid material separately, A is cross-section area, I is second moment of area.

3.2 Uniaxial compression in X direction

Due to symmetry, the vertical struts do not experience any effective load when the structure is uniaxial compressed in X direction. Therefore, the deformation of the structure was solely determined by the oblique struts. It should be note that re-entrant struts QR' (type 1) and $Q''R'$ (type 2) are not mechanically symmetrical, therefore should be considered separately, as shown in Figure 4.

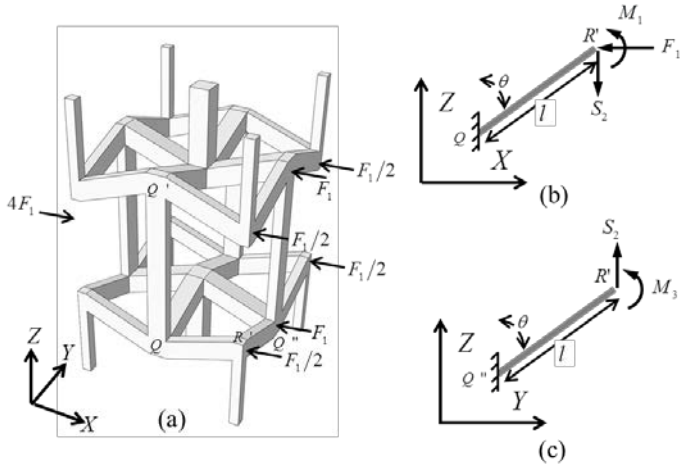

Figure 4: (a) Compressive loading of re-entrant auxetic structure in X direction. (b) Loading condition of the re-entrant strut QR'. (c) Loading condition of the re-entrant strut $Q''R'$.

Then, the displacement of point R' along Z direction $\Delta_{QR'Z}$ can be obtained as

$$\Delta_{QR'Z} = \frac{F_1(l-2\Delta l)^3(\cos\theta-\beta\sin\theta)\sin\theta}{12E_sI} - \frac{F_1(l-2\Delta l)(\sin\theta+\beta\cos\theta)\sin\theta}{E_sA} + \frac{6F_1(l-2\Delta l)(\cos\theta-\beta\sin\theta)\sin\theta}{5G_sA} \tag{8}$$

where β is the ratio of the shear force S_2 to the applied force F_1.

The displacement of point R' along X direction $\Delta_{QR'X}$ can be obtained as

$$\Delta_{QR'X} = \frac{F_1(l-2\Delta l)^3(\cos\theta-\beta\sin\theta)\cos\theta}{12E_sI} + \frac{F_1(l-2\Delta l)(\sin\theta+\beta\cos\theta)\sin\theta}{E_sA} + \frac{6F_1(l-2\Delta l)(\cos\theta-\beta\sin\theta)\cos\theta}{5G_sA} \tag{9}$$

The displacement of point R' along Y direction $\Delta_{Q''R'Y}$ can be obtained as

$$\Delta_{Q''R'Y} = \frac{\beta F_1(l-2\Delta l)^3\sin\theta\cos\theta}{12E_sI} - \frac{\beta F_1(l-2\Delta l)\sin\theta\cos\theta}{E_sA} + \frac{6\beta F_1(l-2\Delta l)\sin\theta\cos\theta}{5G_sA} \tag{10}$$

The Poisson's ratios of the re-entrant auxetic structure compressed in X directions can be readily obtained:

$$\nu_{XZ} = -\frac{\varepsilon_Z}{\varepsilon_X} = -\frac{\dfrac{\Delta_{QR'Z}}{h-l\cos\theta}}{\dfrac{\Delta_{QR'X}}{l\sin\theta}} = -\frac{\Delta_{QR'Z}\sin\theta}{\Delta_{QR'X}(h/l-\cos\theta)} \tag{11}$$

$$\nu_{XY} = -\frac{\varepsilon_Y}{\varepsilon_X} = -\frac{\Delta_{Q''R'Y}}{\Delta_{QR'X}} \tag{12}$$

And the effective modulus of the 3D re-entrant auxetic structure under compression in X direction can be obtained as:

$$E_X = \frac{\sigma}{\varepsilon_X} = \frac{\dfrac{F_1}{(h - l\cos\theta)\, l\sin\theta}}{\dfrac{\Delta_{QR'X}}{l\sin\theta}} = \frac{F_1}{\Delta_{QR'X}\,(h - l\cos\theta)} \tag{13}$$

Table 1 shows the comparison of the analytical model in present study with the experimental results and the analytical model in the work of Yang et al. [10]. From the table it can be seen that, the analytical model in present study matches much better with the experimental results than the analytical models in the work of Yang et al. [10].

Design	v_{ZX}			v_{XZ}			v_{XY}		
	Exp.[Y]	Anal.[Y]	Anal.[P]	Exp.[Y]	Anal.[Y]	Anal.[P]	Exp.[Y]	Anal.[Y]	Anal.[P]
P1	-0.14	-0.47	-0.31	-0.57	-1.50	-0.74	-0.23	-1	-0.43
P2	-0.37	-1.09	-0.55	-0.09	-0.55	-0.34	-0.31	-1	-0.51

Exp.[Y] (abbreviation of experimental results in reference [10]), Anal.[Y] (abbreviation of analytical results in reference [10]), Anal.[P] (abbreviation of analytical results in present study).

Table 1: Comparison of the analytical model in present study
with the experimental results and the analytical model in reference [10]

4 CONCLUSIONS

In this paper, 3D re-entrant auxetic cellular structure that exhibits auxetic behavior in all three principal directions was analytical modeled based on energy methods. When the struts are relative stubby, the overlapping effect of the struts and axial extension or compression should be taken into consideration.

ACKNOWLEDGEMENTS

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