NEW PROOF OF LAMINATE DESIGN BY A PHYSICALLY BASED FAILURE CRITERION

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ABSTRACT

In recent years, the structural analysis of fibre reinforced components has experienced a tremendous improvement. Unfortunately, the development of appropriate failure criteria has not been promoted in the same way. In 1980, Hashin proposed an alternative physically-based failure analysis which could be realized by a considerably increased numerical effort. Recently, Puck developed an extended hypothesis based on Hashin's concept using fundamental elements of the failure criterion of Mohr. Applying this model particularly the three-dimensional state of stress is evaluated in a realistic manner. In contrast to the conventional failure criteria, the new criterion is defined in a coordinate system which rotates with the angle of fracture.

INTRODUCTION

The high potential of fibre reinforced plastics (FRP) for lightweight structures can only be exploited optimally in high performance applications if laminates and component parts are designed in accordance with their loads. This requires reliable calculation concepts which realistically take into account the prevailing state of stress and strain on the one hand and the failure behaviour on the other. With currently existing analytic and numerical methods of calculation, the real stress and strain fields can be determined with sufficient accuracy, as it has been confirmed by numerous experimental studies. In contrast, the corresponding failure analysis in general causes major difficulties.

One reason for this is that the model description of the unidirectional (UD) fibre reinforced layer, which is presupposed in the stress and strain analysis and assumes a homogeneous, anisotropic continuum, is capable of reproducing the laminate behaviour efficiently in terms of the elasticity theory. In contrast, the application of the so-called homogenisation technique in the sense of a "blurred" continuum for the description of the fracture behaviour of UD layers is impermissible in the failure analysis. Naturally, the failure first takes place in the heterogeneous micro-range, so that it is the microstructure which acquires decisive importance. From this point of view, a differentiation between individual fracture types is essential and must be taken into account in the formulation of realistic fracture criteria. Only in this way the non-homogeneous fibre-matrix laminate can be considered with regard to failure.

Although it is possible to verify such physically plausible statements about the fracture event through numerous experiments, so-called generalising fracture criteria are mainly applied in the design of fibre reinforced components. They assume an impermissible view of the "blurred" fibre-matrix continuum in accordance with UD-layer homogenisation. The generalising fracture criteria such as, for example, the quadratic failure criteria of Sacharov, Azzi/Tsai, Tsai/Wu, etc. combine fundamentally different fracture mechanisms in an approximation in the form of one interpolation polynomial.

NEW GENERATION OF FAILURE CRITERIA

A completely different method for the formulation of realistic fracture criteria, taking account of the heterogeneous material structure relevant to the fracture, has been adopted by Hashin [1] and Puck [2, 3]. Here, not only the decisive difference between fibre fracture and inter-fibre fracture is considered, but a fracture angle has been introduced as a free parameter which also characterises further fracture types in the plane parallel to the fibres. It is also taken into consideration that, in accordance with mechanical material behaviour and phenomenological observations, compressive stresses perpendicular to the fibres tend to prevent the formation of an inter-fibre fracture, while corresponding tensile stresses tend to promote inter-fibre fracture. Thus, the new fracture criteria developed by Hashin and Puck, in contrast to the generalising fracture criteria, are founded on physically based phenomena. The first approach in this direction came from Mohr for isotropic materials and has proved very satisfactorily in its description of brittle fracture behaviour.

As early as 1969, Puck [4] pointed out that at least two failure modi ought to be distinguished and described by separate and independent failure criteria: fibre failure (FF) and inter-fibre failure (IFF). A simple failure criterion for fibre fracture is

$$\left(\frac{\sigma_1}{R_{\parallel}^{(\pm)}}\right)^2 = 1 \tag{1}$$

where $R_{\parallel}^{(+)}$ means the longitudinal tensile strength for $\sigma_1 > 0$ and $R_{\parallel}^{(-)}$ the longitudinal strength for $\sigma_1 < 0$. This relation describes clear failure in a fixed plane perpendicular to the fibres.

In contrast, failure involving inter-fibre fracture takes place formally in an initially variable plane parallel to the fibres. In order to permit the characterisation of the different forms and mechanisms of fracture by means of failure criteria of the new generation, it is advisable to introduce a natural coordinate system x_1, x_n, x_t. The orientation of the coordinate system is characterised by the rotation θ around the fibre-parallel x_1 axis (Fig. 1). The fracture angle θ is directly incorporated into the failure criterion (for further details, see [3, 5, 6]). Through the introduction of the rotatable system, not only the usual material stressing can be predicted, but also the fracture directions, which is of crucial importance for local damage analysis.

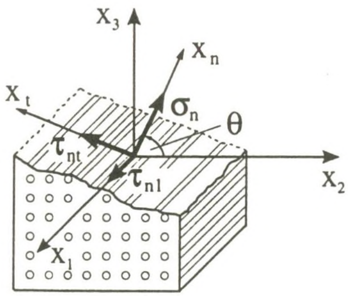

Figure 1. Natural coordinate system in the fracture plane

The corresponding criterion for IFF in general form is [1, 2]

$$f(\sigma_n, \tau_{nt}, \tau_{n1}) = 1 \ . \tag{2}$$

This approach, through logical extension of Mohr's hypothesis, pursues the idea that the stresses in the fracture plane determine the fracture limit of a material (see Fig. 1). This failure hypothesis for inter-fibre fracture is of a more complex nature since it has to combine different forms of failure, while the stresses σ_l, σ_t and τ_{tl}, which act in planes perpendicular to the fibre-parallel fracture plane, have no influence on the fracture event.

A major advantage of the fracture condition with reference to an attached coordinate system is that particularly spatial stress states can be reliably evaluated with regard to failure. In contrast, the conventional generalising fracture criteria, especially in the case of 3-dimensional stress conditions, often produce contradictory results, not at least because they were developed and partly verified for plane, homogeneous fibre laminates. With the increasing applications of thick walled fibre laminate components, however, it has also become necessary to consider the stresses perpendicular to the laminate plane, often critical for failure, in the design process. The same applies to relatively thin walled laminates of modern, highly anisotropic UD layers, as here, critical stress combinations can also occur in planes parallel to laminate plane (i.e. $\theta = 90°$ in Fig. 1) because of the strong deformation inhibitions.

FRACTURE RESISTANCE ANALYSIS USING REALISTIC FRACTURE CRITERIA

For evaluation of the stress conditions calculated or measured on structures, realistic fracture criteria should be applied for the necessary fracture resistance analysis. The dimensioning procedure will be explained using, as an example, the multi-layered cylindrical shell shown below.

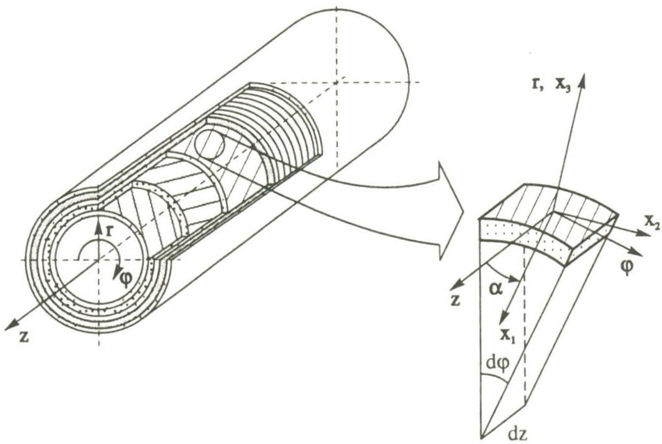

Figure 2. Multi-layered cylindrical shell

In the general layered structure of the multi-layered cylindrical shell (Fig. 2), in contrast to isotropic materials, spatial stress conditions σ_r, σ_φ, σ_z, $\tau_{\varphi z}$ are already induced by rotation-symmetrical loads (tension/compression, torsion, pressure, centrifugal force, etc.), while in the case of non-rotation-symmetrical loads (for example, through pure bending or bending with transverse forces), the remaining shear stresses $\tau_{r\varphi}$, τ_{rz} are also present. Transformation of the stresses into a fibre-adjusted coordinate system for the UD layer always results in a three-dimensional stress state to be evaluated with regard to failure (for details, see [7]).

In the application of conventional fracture criteria, the failure-critical stresses σ_r, $\tau_{r\varphi}$, τ_{rz} often remain unconsidered, although they can be determined using extended laminate theories and modern FEA software. Such gross neglect often leads to unexpected structural failures. The spatial stress state, which is present in the individual UD layers of the layered cylindrical shell (Fig. 2) can be analysed with advantage by means of the new generation of criteria from Hashin/Puck. The stresses are referred to the natural coordinate system.

For the tensile region ($\sigma_n > 0$), Hashin proposes a failure criterion with quadratic terms [1]

$$f(\sigma_n, \tau_{nt}, \tau_{nl}) = \left(\frac{\sigma_n}{R_\perp^{(+)}}\right)^2 + \left(\frac{\tau_{nt}}{R_{\perp\perp}}\right)^2 + \left(\frac{\tau_{nl}}{R_{\perp\parallel}}\right)^2 = 1 \tag{3}$$

and points out that a different failure criterion must be used for the compressive region. In criterion (3), the stresses causing the failure of the laminate are contained in the numerator. The corresponding coefficients $R_\perp^{(+)}$, $R_{\perp\perp}$ and $R_{\perp\parallel}$ are, in order, the strength based on transverse tension, transverse/transverse shear and transverse/longitudinal shear.

Using the tensor transformation, the stresses σ_n, τ_{nt} and τ_{nl} can be expressed in terms of the stresses σ_2, σ_3, τ_{23}, τ_{31}, τ_{23} and the angle $(x_2, n) = \theta$, so that the general form of (3) is as follows

$$g(\sigma_2, \sigma_3, \tau_{23}, \tau_{31}, \tau_{21}, \theta) = 1 \ . \tag{4}$$

The failure of one UD layer takes place on that fracture surface marked with the fracture angle $\theta = \theta_{fp}$ (fp: failure plane) in Fig. 1. This is where the function in (4) reaches its maximum. The stress combination leading to the fracture can only be determined when the fracture angle is known. Usually numerical methods having to be employed for the solution of the extreme value problem [3, 8]. Because of the mathematical difficulties, Hashin did not follow up his idea.

Only the rapid developments of the last few years in the computing sector and Puck's further considerations on the basis of the Hashin hypothesis have confirmed that the realistic criteria for the practical design of fibre laminate structures have today already reached their potential maturity for application. In 1992, Puck proposed extended theorems for improved strength analysis with regard to inter-fibre fracture. They are based on the following fracture hypothesis [2]:

- **Tensile region ($\sigma_n \geq 0$)**: When a fibre-parallel fracture surface appears, the fracture is caused jointly by the transverse tensile stress σ_n and the two shear stresses τ_{nt} and τ_{nl} acting in the fracture plane (at the moment of fracture).

$$\left(\frac{\sigma_n}{R_\perp^{(+)}}\right)^2 + \left(\frac{\tau_{nt}}{R_{\perp\perp}^A}\right)^2 + \left(\frac{\tau_{nl}}{R_{\perp\parallel}}\right)^2 = 1 \tag{5}$$

- **Compression region ($\sigma_n < 0$)**: A transverse shear stress acting on the fracture plane makes no contribution towards initiating the fracture. On the contrary, it impedes the shear fracture caused by the shear stresses τ_{nt} and τ_{nl} by giving rise to additional resistance to shear fracture; this resistance increases as a function of the compressive stress.

$$\left(\frac{\tau_{nt}}{R_{\perp\perp}^A - p_{\perp\perp}\sigma_n}\right)^2 + \left(\frac{\tau_{nl}}{R_{\perp\parallel} - p_{\perp\parallel}\sigma_n}\right)^2 = 1 \tag{6}$$

The proportionality factors $p_{\perp\perp}$ and $p_{\perp\parallel}$ in Equation (6) cover the influence mentioned of the normal stress σ_n on failure in the compressive region and can be obtained from experimental studies. The action plane resistance $R_{\perp\perp}^A$ (A: action plane) which occurs in both of the above fracture conditions in combination with the transverse/transverse shear stress τ_{nt} must not (as, for example in the Hashin criterion (3)) be replaced by the corresponding strength $R_{\perp\perp}$. The reason for this is that in UD laminates the shear stress τ_{nt}, unlike the tensile stress σ_n and the shear stress τ_{nl}, causes no failure in its action plane (for more details, please see [3, 6]).

The fracture surface in accordance with fracture conditions (5) and (6) is shown in Figure 3. Meanwhile, Puck has proposed further theorems for an improved strength analysis which permit determination of the fracture angle with less mathematical effort. Which of these theorems best reproduce the failure of UD laminates is at present being studied using special testing technology in the framework of a research project [9].

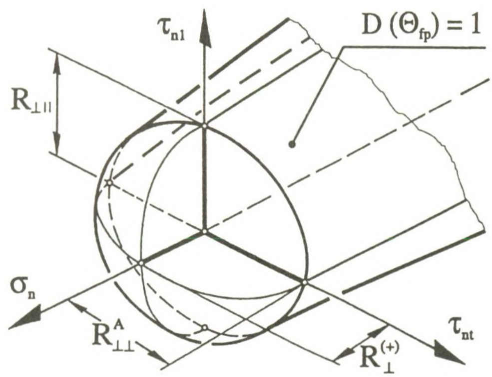

Figure 3. Fracture surface with reference to natural coordinate system

FAILURE BEHAVIOUR UNDER HYDROSTATIC PRESSURE

Experimental difficulties arise in the verification of the conventional and new failure criteria, especially in the compressive region. Firstly this is due to the high forces needed here and, secondly, the fracture angle often is not clearly determinable as the fracture surfaces are pressed into one another. Failure as a result of multiaxial compressive stress combinations is therefore the subject of intensive studies. As an example of the verification of the new fracture condition in accordance with Equation (6), tangentially wound tubular test samples made of GFRP (internal diameter: 40 mm, thickness: 2 mm) were subjected to a uniform, hydrostatic pressure of p = 6000 bar. After this compressive test, no failure of the wound samples was detected!

Calculation of the stresses is carried out with the help of the theorems for thick walled cylindrical shells (see e.g. [7]). The stress curve over the radius, independent of the longitudinal coordinate, is shown in Figure 4. The extreme values occur at the internal radius. Because of the technologically determined winding angle deviation of 2° in the deposition of the fibre, additional small shear stresses $\tau_{\varphi z}$ arise which are taken into account below, but which have no influence on the result of the test.

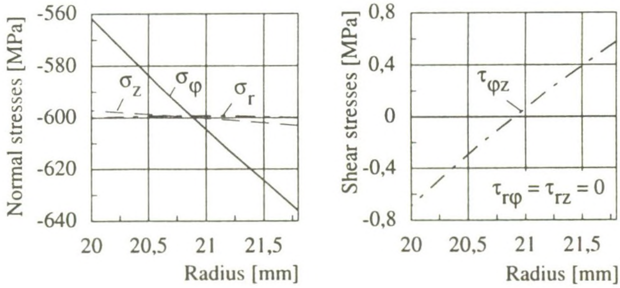

Figure 4. Curve of normal stresses in a hydrostatic loading condition p = 6000 bar

For the application of criterion (6), the following standard properties for GFRP laminates are assumed [3]:

$$R_{\perp\perp}^{A} = 81.5\,\text{MPa} \quad p_{\perp\perp} = 0.1$$
$$R_{\perp\parallel} = 80\,\text{MPa} \quad p_{\perp\parallel} = 0.2\,.$$

From Puck's failure criterion, the resultant fracture risk $D(\theta_{fp})$ for the most highly stressed inner mantle region is smaller than 0.1 % (at a fracture angle of $\theta_{fp} = 0°$) so that, in accordance with the experimental results, no failure of the laminate can yet occur. In contrast, from the failure criterion of Tsai/Wu for the same evaluation point it follows that the fracture risk $D(\theta_{fp})$ is approx. 400 %, indicating a fourfold exhaustion of the material reserves.

SOME DEFICIENCIES OF THE GENERALISING FRACTURE CRITERIA

When generalising fracture criteria are used, the difference between the significant fracture mechanisms in fibre laminates is excluded from the start. Rather, it is assumed purely theoretically, without consideration of the true material structure, that the fracture surface is a continuously differentiable function, fixed in the stress space by only a few experimentally verified interpolation points. Between the corner points, the surfaces are approximated by means of simple interpolation polynomials and designated as fracture surfaces. A very widespread criterion of this type is the quadratic failure criterion from Tsai/Wu (1971) which had already been introduced ten years before by Sacharov (cf. [10, 11]):

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \qquad (i,j = 1,\ 2,\ \ldots\ 6)\ . \tag{7}$$

A comparison between the conventional failure criteria and the new criterion published by Puck in 1969 [4] is presented for the stress plane σ_1, σ_2 in Figure 5.

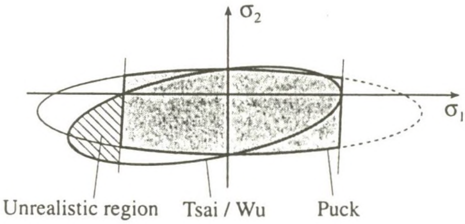

Figure 5. Schematic comparison of the Tsai/Wu and Puck criteria

It can be clearly seen that, in the case of the Puck criterion, the stresses in the fibre direction may not exceed the specified limit values, as uncoupled theorems come into application here for fibre fracture and inter-fibre fracture. Particularly great discrepancies between the two criteria result in the compression/compression region, as in accordance with the Tsai/Wu criterion, the safety-relevant region can be strongly influenced by the individual basic strengths. Thus, for example, a reduction of the tensile strength perpendicular to the fibres leads to a practically inexplicable rise in the fracture reserves for $\sigma_1, \sigma_2 < 0$. Such a situation is sketched in Figure 6 and has already been frequently quoted as proof of the physical deficiencies of generalising criteria (for more on this subject, see the numerous publications of Hart-Smith, e.g. [12]).

A further problem in the application of classical criteria is represented by the interaction coefficient F_{12} which must be within specified limits in accordance with $F_{12}^2 > F_{11}F_{22}$ in order that the fracture body remains "closed". For the experimental determination of the true values of F_{12}, glass fibre reinforced plastics were studied with off-axis loading in both tensile and shear tests.

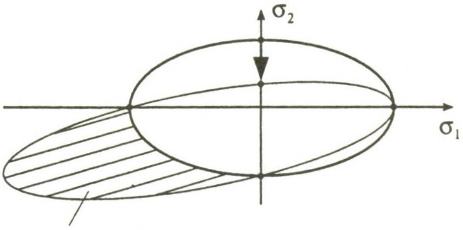

Figure 6. Influence of transverse tensile strength on the failure surface using Tsai/Wu criteria

The significant results are summarised in Figure 7. The consequence is that in both test types the coefficient F_{12} assumes for almost all fibre orientations values, which indicates an unclosed fracture body. This is physically not accurate. In practice, this coefficient is often assigned a value without experimental verification, for example zero or $(F_{11}F_{22})^{0.5}/2$, with which the fracture body is forcibly closed, at least in theory.

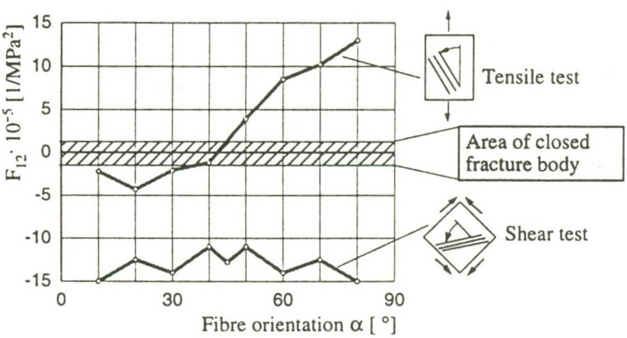

Figure 7. Experimentally determined strength coefficient F_{12} from off-axis tests

CONCLUSION

The application of realistic failure criteria for the UD layers in laminate design is a precondition for the successful use of laminated components in lightweight structures. As the non-homogeneous, anisotropic layer structure naturally has a major influence on fracture behaviour, the special structural relationships presented here should always be taken into account in the formulation of adjusted fracture criteria. This statement based on materials mechanics forms the basis of the new IFF fracture criterion according to Hashin and Puck. For the first time, with the introduction of the fracture angle into the criterion, both the most heavily loaded fibre-parallel layer plane and, at the same time, the type of fracture can be identified, opening new possibilities for the analysis of the successive fracture processes of multilayered laminates. First experimental studies confirm that the theorems developed by Puck, in particular, reproduce very accurately the actual fracture behaviour of UD laminates, even in the case of a 3-dimensional stress conditions.

In contrast, the generalising fracture criteria lead to major inconsistencies, not only in their experimental verification, but, in particular, also from a materials engineering viewpoint. These include, among others,

- impermissible "homogenisation" of the fibre/matrix laminate
- no differentiation between fibre fracture and inter-fibre fracture
- disproportionately high fracture reserves in the compressive strength region of the fibre

– unrealistic geometrical distortions of the whole fracture body as a result of altering only one strength value
– large discrepancies between results obtained experimentally and using generalising criteria in 3-D stress state, especially in the compressive region.

Through the application of the newly developed failure criteria, the weaknesses of the generalising criteria can be avoided, the lightweight construction potential of modern fibre laminates can be much better exploited and new design concepts can be developed. In the framework of a cooperative research project [9], evidence is currently being gained by means of special testing technology that the criterion according to Puck realistically describes the fracture behaviour of the UD base layer for all stress planes.

ACKNOWLEDGEMENTS

For scientific discussion of this paper, we especially thank Prof. (em.) Dr.-Ing. A. Puck. Financial support of the project [9] by the German Ministry of Science and Technology (BMFT) is gratefully acknowledged. Furthermore, we wish to thank all project partners:

MAN: Dr.-Ing. habil. R. Cuntze, Dipl.-Ing. B. Szelinski, Dipl.-Ing. R. Deska
GHK: Mrs. Prof. Dr. rer. nat. habil. R. Jeltsch-Fricker, Mrs. Dr. rer. nat. S. Meckbach
IKV: Prof. Dr.-Ing. W. Michaeli, Dipl.-Ing. D. Huybrechts
RCP: Prof. Dr.-Ing. habil. R. Rackwitz, Dr.-Ing. St. Gollwitzer

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