# A CONTINUUM DAMAGE MODEL FOR COMPOSITE LAMINATED STRUCTURES SUBMITTED TO STATIC AND FATIGUE LOADINGS

P. Nimdum[1*], J. Renard[1*]
[1] Mines-Paris Tech, CNRS UMR 7633, BP 87, F-91003 Evry, Cedex
* Corresponding authors (pongsak.nimdum@ensmp.fr, jacques.renard@ensmp.fr)

**Keywords**: *thick composite, damage model, matrix cracking, delamination onset*

## Abstract

This study is mainly focused on two major types of damage: intra-laminar ply cracking and inter-ply delamination in fiber-reinforced composite under static and fatigue loading. Off-axis and angle-ply non woven laminate have been used to study matrix cracking. Damage evolution based on continuum damage mechanics is considered for prediction intra-laminar ply cracking. While, angle-ply woven laminate is used to study the delamination. The delamination onset criterion based on average stress has been proposed. Identification of the different parameters of damage evolution model of matrix cracking and delamination onset has been made with classical rectangular specimen. Validation was made with static and fatigue tests performed on non woven and woven laminates with drilled circular hole. The numerical predictions are in good agreement with experimental results.

## 1. Introduction

Thick composite laminated structures able to support significant efforts, are more and more used for engineering structural parts. Then it is necessary to consider the ability of such laminates to resist from damage development, the consequence of which is mechanical degradation of properties (stiffness decrease). Thus, it requires appropriate design tools to prevent from damage evolution and predict the influence of damage on mechanical properties.

Damage mechanisms up to failure are rather complex in composites laminates. One reason is that several damage phenomena (matrix cracking, delamination, fiber breaking, fiber/matrix interface debonding …) are acting alone or coupled.

These last remarks explain the different philosophy associated with these two types of damage: (i) considering ply and matrix cracking, a damage coupled behaviour model has been proposed based on an understanding of physical mechanisms [1-3]. As this type of damage is not so critical for the structure, we consider its possible evolution during calculation and prediction of life-time. We can then simulate the influence of ply cracking on the mechanical properties of the structure, (ii) considering delamination an onset criterion has been proposed [4]. As this damage type could be critical for a structure, the objective in this case is rather to design a structure by avoiding delamination.

The objective in this study is to investigate the matrix cracking using damage mechanics and to predict initiation of delamination in woven and non woven laminate during static and fatigue loading.

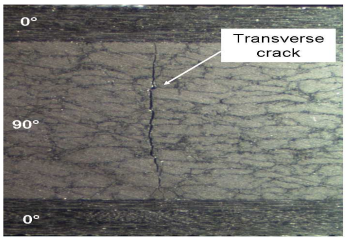
## 2. Experimental procedure

Static and fatigue tension-tension loading were performed in woven and non-woven carbon fiber-reinforced epoxy laminates. In fatigue case, the different maximum applied stress, $\sigma_{\max}$, which are less than the ultimate tensile stress ($\sigma_R$) is applied. The load ratio, $R$, and the frequency (f) is 0.1 and 1 Hz, respectively. All tests are performed at room temperature. The off-axis and angle-ply non-woven laminates ($(0°_3,90°_n)_s$, $(0°_3,\pm45°_3)_s$ and $(0°_3,\pm55°_3)_s$ ) have been tested in order to study different initiation and evolution of mode of matrix cracking. Whereas, the angle-ply woven laminate ($(0°,\pm20°)_s$, $(0°,\pm20°_2)_s$, $(0°,\pm30°)_s$ and $(0°, \pm30°_2)_s$) is to study the delamination onset.

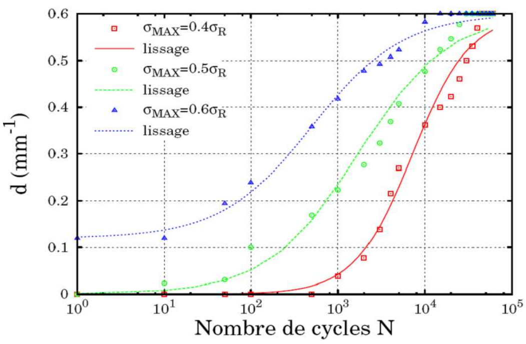
## 3. Damage mechanism

### 3.1 Matrix cracking

In the similar way to the previous study, the experimental results show that matrix cracking is a diffuse damage on off-axis laminate (Fig.1). Crack density evolution is strongly dependent on maximum applied stress. However, for a given stacking sequence, the difference of maximum applied stress level gives the same saturation density. We found that a good correlation is observed between static and fatigue loading for matrix damage.




(a)




(b)

Fig.1. (a) Transverse crack and (b) crack evolution density in $(0°_3,90°_n)_s$ laminate under cyclic loadings.

### 3.2 Delamination onset

We now consider the case of the woven angle-ply laminates. The experimental results illustrate that the delaminations are not straight (plan) but bended. After the interlaminar delamination appeared, we investigate on stiffness degradation and find that the modulus decrease can be divided into three stages (Fig.2): (i) initial region (stage I) with a slightly decrease stiffness reduction of about 2.5%, and then, (ii) an intermediate region (stage II), in which an additional about 10% stiffness reduction occurs in an approximately linear fashion and (iii) the final region (stage III) with a rapid decrease of stiffness (about 40%), and then the stiffness reduction become unstable and lead to the final failure of specimen. In the same manner with matrix cracking, the delamination mechanism is a good correlation between static tensile and fatigue loading.

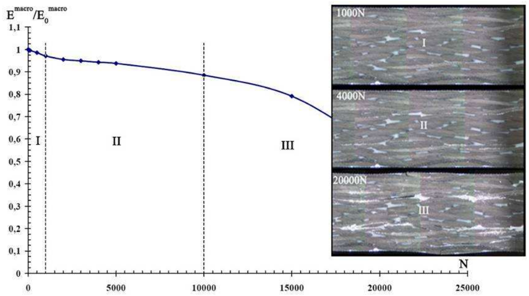



Fig.2. Stiffness degradation as a function of number of cycles

In order to propose the criterion for prediction the onset of delamination under fatigue loading, the relationship of $\sigma_{\max}$ , onset of delamination stress under tensile loading test ( $\sigma_{\text{onset}}$ ) and the number of cycles onset ( $N_a$ ) is investigated as shown in Fig.3. This nonlinear relation can be expressed as:

$$\frac{\sigma_{\max}}{\sigma_R} = (K_1)^{-(N-1)^{K_2}} \tag{1}$$

where $K_1$ and $K_2$ are two constant parameters, these parameters are independent on i.e. the stacking sequence and number of plies (thickness) but depend on the composite material study.

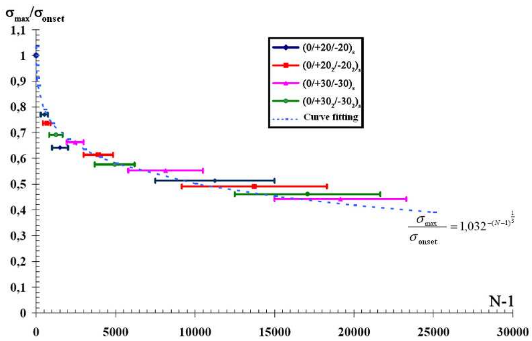



Fig.3. A number of cycles to delamination onset ( $N_a$ ) in different stacking sequences

## 4. Numerical

### 4.1 Damage mechanic for matrix cracking

In this study, we choose to develop a damage model within the framework of the Damage Mechanics. The model is written at the scale of the ply and describes the multiplication of cracks and loss of rigidity caused by damage. The ply cracking is described by two internal variables: the first one is a scalar which characterizes the damage state of the materials, *f*. The second one is a vector which describes the direction of damage, *U*,[3,5]:

$$\vec{V}=\begin{Bmatrix} V_T(\alpha,m,r) \\ V_N(\alpha,m,r) \\ 0 \end{Bmatrix} = f(\alpha)\times\begin{Bmatrix} U_T(m,r) \\ U_N(m,r) \\ 0 \end{Bmatrix} \quad (2)$$

In the quasi-static loading, the free energy state function $\psi$ is written in order to describe the damage evolution law:

$$\begin{aligned}\psi=\psi(\varepsilon,\vec{V}(\alpha,m,r))=\varphi^{0}(\varepsilon)+\varphi^{T}(\varepsilon,V_T)\\ +\varphi^{N}(\varepsilon,V_N)+\varphi^{NT}(\varepsilon,V_N,V_T)\end{aligned} \quad (3)$$

with, $A$ is dual variable of α:

$$A=\frac{\partial\psi(\varepsilon,\vec{V}(\alpha,m,r))}{\partial\alpha} \quad (4)$$

And threshold in static loading is written as:

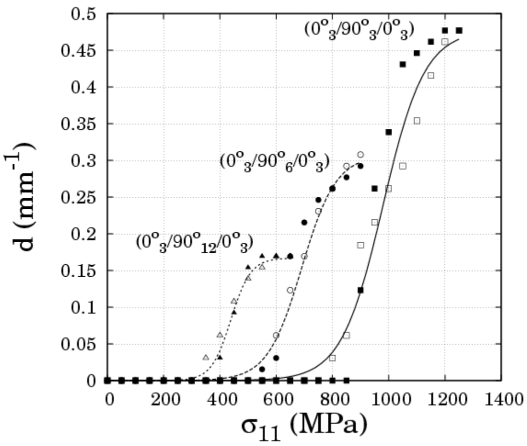
$$c=A^{C}(\alpha,m,r)-A(\varepsilon,\alpha,m,r)\le 0 \quad (5)$$

Finally, we can simplify the damage evolution law in static case as:

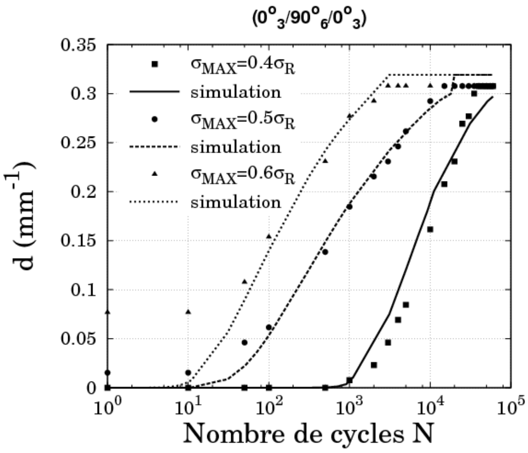
$$\begin{aligned}d\alpha=\frac{-\dfrac{\partial^2\psi}{\partial\varepsilon\partial\alpha}\partial\varepsilon}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}+\frac{\left(\dfrac{\partial^2\psi}{\partial\alpha\partial m}-\dfrac{\partial A^c}{\partial m}\right)dm}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}\\ +\frac{\left(\dfrac{\partial^2\psi}{\partial\alpha\partial r}-\dfrac{\partial A^c}{\partial r}\right)dr}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}\end{aligned} \quad (6)$$

$$\begin{aligned}d\alpha=\frac{-\dfrac{\partial^2\psi}{\partial\varepsilon\partial\alpha}\partial\varepsilon}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}+\frac{\dfrac{\partial A^c}{\partial N}dN}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}\\ +\frac{\left(\dfrac{\partial^2\psi}{\partial\alpha\partial m}-\dfrac{\partial A^c}{\partial m}\right)dm}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}+\frac{\left(\dfrac{\partial^2\psi}{\partial\alpha\partial r}-\dfrac{\partial A^c}{\partial r}\right)dr}{\dfrac{\partial^2\psi}{\partial\alpha^2}-\dfrac{\partial A^c}{\partial\alpha}}\end{aligned} \quad (7)$$

In fatigue problem, we assume that the damage phenomenon is physically and geometrically identical under quasi-static. Then, the damage evolution law (Eq. (6)) can be rewritten to take into account both quasi-static and fatigue loading case as in equation (7).




(a)




(b)

Fig.4. Numerical results for identification of static (a) and fatigue loading (b)

As the previous study [3], we suppose that $m$ has a more significant influence than $r$ on the damage evolution. As a result, the critical threshold, $A^{C}$, is written as:

$$A^C(\alpha, m, N) = -b(m,N).e^{\left\{\frac{\ln\left(-\ln\left(1-\frac{\alpha}{c(m,N)}\right)\right)}{a(m,N)}\right\}} \quad (8)$$

Then, $A^C$ must be identified using experimental data. We note that the critical threshold for N=1 is the static case. Fig. 4 shows the numerical identification results in static and fatigue case.

### 4.2 Delamination onset criterion

In order to predict the delamination onset, the static and fatigue criterion are proposed. In order to avoid the mesh-dependent of FE approximation for delmaination onset prediction, many non-local stress criteria have been developed. We can classify them into two categories. The first category of criteria [6,7] is based on the average stress. Then, the second categories of criterions represented the gradient of stress singularities using the weight function which allow to evaluate the weight changes in quantities [8,9]. In this study, we have been selected based on the average stress, $\tilde{\bar{\sigma}}_{ij}$ and $\bar{\sigma}_{ij}$ , with and without their gradients in the vicinity of singular point, respectively The gradient effect is taken into account by the first order derivative. The non-local stresses are then defined as:

$$\tilde{\sigma}_{ij} = \begin{cases} \sigma_{ij} & \text{if } y = 0 \\ \sigma_{ij} + h \cdot \left(\frac{\partial \sigma_{ij}}{\partial x} + \frac{\partial \sigma_{ij}}{\partial y} + \frac{\partial \sigma_{ij}}{\partial z}\right) & \text{if } y > 0 \end{cases} \quad (9)$$

where $\boldsymbol{h}$ is characteristic length for taking into account the gradient effect. Note that $\boldsymbol{h}$ depends on the constituents, their geometries (lamina, woven-fabrics) of composites studied. However, to due to the main aim of this paper, we will investigate the delamination onset at interface of adjacent layers. As a result, we can suppose that the gradient effects have been only evaluated along the interface. The Eq. (9) can then be rewritten as:

$$\tilde{\sigma}_{ij} = \begin{cases} \sigma_{ij} & \text{if } y = 0 \\ \sigma_{ij} + h \cdot \frac{\partial \sigma_{ij}}{\partial y} & \text{if } y > 0 \end{cases} \quad (10)$$

The criterion is proposed to prediction the delamination onset under static and fatigue loading. We assume that each ply remains undamaged until the onset of delamination and the interlaminar tensile and shear strength of the interface, which consist of three modes (I, II, III) and are determined by static tests, are damaged under fatigue loading. This degradation is related as a function of $\boldsymbol{\sigma}_{\max}$ , $\boldsymbol{R}$ , $\boldsymbol{N}$ and frequency (f) . The delamination onset criterion under fatigue loading is then introduced as:

$$\left(\frac{\langle F_3\rangle^+}{Y_T^{\text{Fatigue}}(N,\mathrm{f},R,\sigma_{\max})}\right)^2 + \left(\frac{|F_1| + k_1\langle F_3\rangle^-}{S_1^{\text{Fatigue}}(N,\mathrm{f},R,\sigma_{\max})}\right)^2 + \left(\frac{|F_2| + k_2\langle F_3\rangle^-}{S_2^{\text{Fatigue}}(N,\mathrm{f},R,\sigma_{\max})}\right)^2 = 1 \quad (11)$$

In this study, R and f are constant at 0.1 and 1 Hz, respectively, to be able to simplify the equation above and to be rewritten as:

$$\left(\frac{\langle F_3\rangle^+}{Y_T \cdot f_1(N,\sigma_{\max})}\right)^2 + \left(\frac{|F_1| + k_1\langle F_3\rangle^-}{S_1 \cdot f_2(N,\sigma_{\max})}\right)^2 + \left(\frac{|F_2| + k_2\langle F_3\rangle^-}{S_2 \cdot f_3(N,\sigma_{\max})}\right)^2 = 1 \quad (12)$$

where $\boldsymbol{Y}_{\boldsymbol{T}}^{\text{Fatigue}}$ , $\boldsymbol{S}_1^{\text{Fatigue}}$ and $\boldsymbol{S}_2^{\text{Fatigue}}$ represent the intralaminar tensile (mode I) and shear (mode II and III) strengths under cyclic loading, respectively, while $\boldsymbol{Y}_{\boldsymbol{T}}$ $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$ de II and III) are the intralaminar tensile (mode I) and shear (mode II and II) under quasi-static, respectively. $f_1(N,\sigma_{\max})$ , $f_2(N,\sigma_{\max})$ and $f_3(N,\sigma_{\max})$ are the degradation function of interlaminar strengths as mode I, II and III, respectively. Assume that three modes are the same rate of degradation. In the same way as for static loading, two modes of interlaminar shear strength were assumed to be identical $(\boldsymbol{S}_1 = \boldsymbol{S}_2)$ . The Eq. (12) can then be rewritten as:

$$\left(\frac{\langle F_3\rangle^+}{Y_T}\right)^2 + \left(\frac{|F_1| + k_1\langle F_3\rangle^-}{S}\right)^2 + \left(\frac{|F_2| + k_2\langle F_3\rangle^-}{S}\right)^2 = \left(f(N,\sigma_{\max})\right)^2 \quad (13)$$

To determine the degradation function, woven angle-ply laminate chosen are represented the predominant shear mode (III) to allow to neglect both mode I and mode II. Due to undamaged and in-plane isotropic behavior assumption until the delamination onset, it is useful to be able to determine the relation in Eq. (10) by the experimental results (macroscopic stress), (Fig. 13). The degradation function of interlaminar strength under fatigue loading is given as:

$$\left(\frac{|\sigma|}{S}\right)_{\text{Non local}} = \left(\frac{\sigma_{\max}}{\sigma_{\text{onset}}}\right)_{\text{Macroscopic}} = f(N) = 1{,}032^{-(N-1)^{\frac{1}{3}}} \qquad (14)$$

In order to apply this criterion in Eq. (13) to circular-hole specimens, the interlaminar tensile and shear strengths in static case are modified. The transformation of coordinates as a function of angle of $\theta$ is used. $\theta$ is defined as illustrated in Fig. 5. By substituting Eq. (14) into Eq. (13), then, the delamination onset criterion during fatigue loading applied to circular-hole specimens is therefore shown in Eq. (15).

$$\frac{\left(\langle F_3\rangle^{+}\right)^2}{Y_T^2} + \frac{\left(\langle F_3\rangle^{-}\right)^2}{S^2/(2k^2)} + \frac{|F_2|\langle F_3\rangle^{-}(\cos\theta+\sin\theta)}{S^2/(2k)} + \frac{|F_1|\langle F_3\rangle^{-}(\cos\theta-\sin\theta)}{S^2/(2k)} + \frac{(F_2)^2}{S^2} + \frac{(F_1)^2}{S^2} = \left(1{,}032^{-(N-1)^{\frac{1}{3}}}\right)^2 \qquad (15)$$

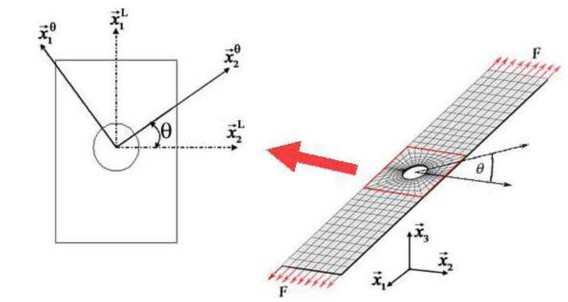


Fig.5. Configuration of circular-hole specimens

## 5. Validation in circular-hole specimen

In order to validate the matrix cracking model and delamination onset criterion in previous section, the experimental tests and numerical simulations in circular-hole specimens under static and fatigue loadin is considered. The circular hole specimens polished prior to testing are observed at different stress and number of cycles level with the mirror which allow its to rotate around the axe. The angle observation ($\alpha$) is defined to respect the applied loading direction, $\alpha = \theta + 90°$.

### 5.1 Matrix cracking

By experimental observation, we find that the matrix cracking is observed between +45° and 135° at free edge in circular-hole specimen. This result is a good correlation with numerical prediction as shown in Fig. 6.

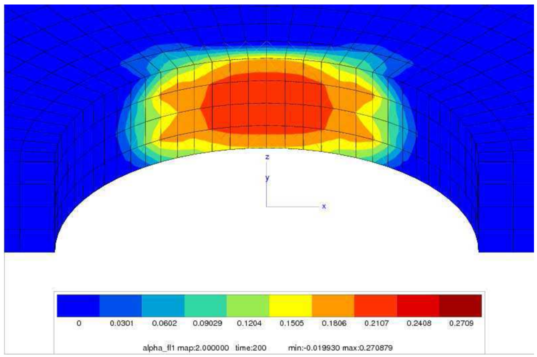
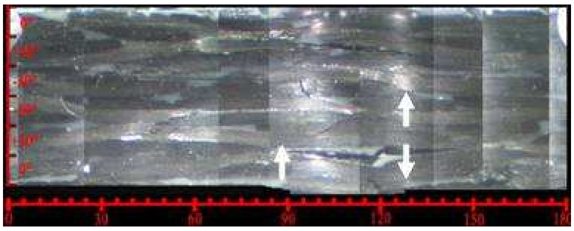



Fig.6. Numerical results for (0°3/90°6/0°3) laminate under σmax=0.4 $\sigma_R$ fatigue loading

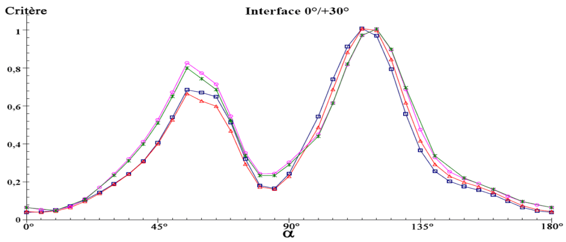
### 5.2 Delamination onset

The numerical results of all stacking sequences of a laminate composite show the good accuracy of location predictions to delamination onset compared with the experimental results, as example illustrate in Fig. 7. The two approaches criterion, with and without gradients, predict to the same location of delamination onset.

For the prediction of number of cycles to delamination onset, there is a good correlation between the numerical results and experimental observations. The close predictions obtained for $0°/20°$ and $0°/30°$ interfaces while the slight prediction error of $+20°/-20°$ and $+30°/-30°$ interfaces can be found (Table 1). We suppose that the slight prediction error is caused by a typical mode III-type of delamination onset which is dominant in these interfaces. Consequently, this mode of delamination onset is more difficult to observe due to the closing mode. To overcome this problem, high accuracy observation techniques like acoustic emission are necessary.



(a)




(b)

Fig.7. Delamination onset in $(0°,\pm 30°)_s$ laminate at free-edge circular hole: (a) experimental observed; (b) numerical prediction at interface $0°/30°$

Table 1
Numerical prediction vs. experimental result of delamination onset stress

| Interfaces | Number of cycles to delamination onset $(N_d)$ | Numerical results | |
|---|---|---|---|
| | | h = 0.0 | h = 0.01 |
| $0°/+20°$ | 2000 – 3000 | 3280 | 3800 |
| $+20°/-20°$ | 260 - 700 | 150 | 600 |
| $0°_2/+20°_2$ | 1660 - 3050 | 3200 | 3700 |
| $+20°_2/-20°_2$ | 125 - 450 | 0 | 18 |
| $0°/+30°$ | 340 – 800 | 250 | 450 |
| $+30°/-30°$ | 215 – 600 | 17 | 135 |
| $0°_2/+30°_2$ | 587 - 1250 | 370 | 600 |
| $+30°_2/-30°_2$ | 50 – 220 | 0 | 0 |

## 6. Conclusion

An experimental investigation and FEM in two damage, matrix cracking and delamination for carbon/epoxy angle-ply fabric and non fabric laminates. Matrix cracking is the main damage for off-axis laminate. On the other hand, the delamination is significant damage in angle-ply laminate.

The continuum damage model, damage evolution law and delamination onset criteria, are proposed under static and fatigue. The numerical prediction in circular-hole specimen gives a good correlate with experimental result under static and fatigue loading.

## ACKNOWLEDGMENTS

We gratefully acknowledge ADEME (Agence de l'Environnement et de la Maîtrise de l'Energie) for support from ARMINES Centre des Matériaux, in the project LICOS and ALSTOM.

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