ESTIMATION OF MECHANICAL BEHAVIOR OF BRAIDED COMPOSITES BASED ON MESH SUPERPOSITION METHOD

D. Ohyama[1*], T. Kurashiki[1], Y. Watanabe[1], Y.Fujita[1], and M. Zako[2]
[1] Dept. of Management of Industry and Technology, Osaka University,
2-1, Yamada-oka, Suita, Osaka, Japan,
[2] Center of Advanced Career Development, Osaka Univ., Osaka, Japan
* d-ohyama@mit.eng.osaka-u.ac.jp

Keywords: *mesh superposition method, tetrahedral element, hexahedral element, braided composite, periodic boundary condition,*

1. Introduction

Braided composites have been used widely in many engineering fields where the high mechanical characteristics and light weight properties were important. In order to estimate the mechanical characteristics of the braided composites, the numerical technique is very effective to reduce the testing and production costs. However, if the architecture of fiber bundle is more complex such as 3-D braided composite and woven angle ply laminate, it is difficult to generate FE models. Therefore, in order to solve the modeling problem, an individual modeling method of fiber bundle parts and resin for the FE analysis of braided composite is proposed based on the mesh superposition method. The concept of mesh superposition method was developed by J. Fish [1]. The most interesting point of the mesh superposition method is that the method enables to estimate the mechanical behaviors of two FE models considering their interaction even though the models are generated individually.

In the previous study [2], the global mesh and local mesh were generated by using the hexahedral elements. However, in order to apply to the various complicated structure of complicated fiber bundle structure, tetrahedral element will be needed.

Therefore, in this research, a new mesh superposition method is proposed by applying the tetrahedral elements as a local mesh and the hexahedral elements as a global mesh.

2. Numerical method

The stiffness equation based on the conventional mesh superposition method is shown in Eq.(1)-(4).

$$\begin{bmatrix} [K^G] & [K^{GL}] \\ [K^{LG}] & [K^L] \end{bmatrix} \begin{Bmatrix} \{d^G\} \\ \{d^L\} \end{Bmatrix} = \begin{Bmatrix} \{f^G\} \\ \{f^L\} \end{Bmatrix} \tag{1}$$

$$[K^G] = \int_{\Omega^G} [B^G]^T [D^G][B^G] d\Omega + \int_{\Omega^L} [B^G]^T [D^L][B^G] d\Omega \tag{2}$$

$$[K^L] = \int_{\Omega^L} [B^L]^T [D^L][B^L] d\Omega \tag{3}$$

$$[K^{GL}] = \int_{\Omega^L} [B^G]^T [D^L][B^L] d\Omega \quad , \quad [K^{LG}] = [K^{GL}]^T \tag{4}$$

Where K^G is Global matrix, K^L is Local matrix and K^{GL} is Global-Local matrix to consider the interaction between Global and Local.

The calculation of K^G is very important because the integration region in this section is hollow and it is difficult to calculate the Gaussian integral with the conventional FEM. Therefore, the approximation method like Eq.(5) is used by the conventional mesh superposition method.

$$\begin{aligned} [K^G] &\approx \int_{\Omega^G} [B^G]^T [D^G][B^G] d\Omega + \int_{\Omega^L} [B^G]^T [D^G][B^G] d\Omega \\ &= \int_{\Omega} [B^G]^T [D^G][B^G] d\Omega \end{aligned} \tag{5}$$

However, this method has some problems such as the numerical accuracy in case that mechanical property between global and local mesh are quite different or each scale of global and local mesh are quite different. In order to solve the problem, the calculation of K^G is modified by using the Eq.(6) proposed by Zako [3].

$$\begin{aligned} [K^G] = &\int_{\Omega} [B^G]^T [D^G][B^G] d\Omega + \int_{\Omega^L} [B^G]^T [D^L][B^G] d\Omega \\ &- \int_{\Omega^L} [B^G]^T [D^G][B^G] d\Omega \end{aligned} \tag{6}$$

The verification of numerical accuracy had been investigated with hexahedral elements as global and local mesh [3]. However, the mesh superposition method with the tetrahedral elements as a local mesh and the hexahedral elements as a global mesh has not been verified.

3. Verification of mesh superposition method

In order to verify the numerical accuracy of mesh superposition method, the convenient model in Fig.1 was prepared. The global mesh of a cube model which means the isotropic matrix resin parts was generated by hexahedral elements, and the local mesh which means an isotropic inclusion was prepared by tetrahedral elements. The numerical simulation under tensile loading in Fig.1 has been carried out in the cases of various elastic modulus of inclusion (*Ei*) and matrix (*Em*).
The center node of inclusion parts was defined the evaluating point in Fig.1, and the displacement (*d*) of the node was estimated under tensile loading.

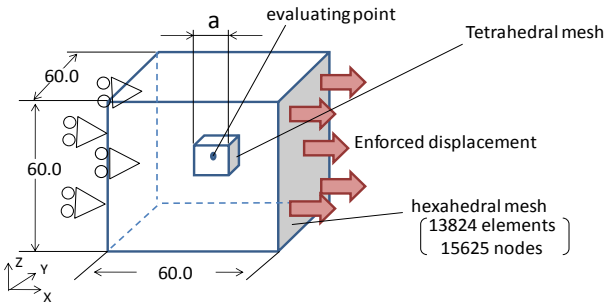


Fig.1 Boundary condition under tensile loading

The displacement of the evaluating point is compared with the numerical results (d_{fine}) by using a fine mesh model composed of inclusion and matrix without mesh superposition method. In order to estimate effects of the size of local model on mechanical properties, three kinds of the local model size (*a*) are prepared in Fig.2.

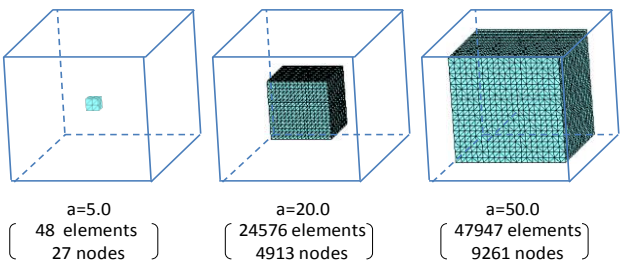


Fig.2 Three kinds of the local model

The numerical error between the fine mesh and the mesh superposition was estimated by Eq.(7).

$$E = \frac{d_{fine} - d}{d_{fine}} \times 100[\%] \qquad (7)$$

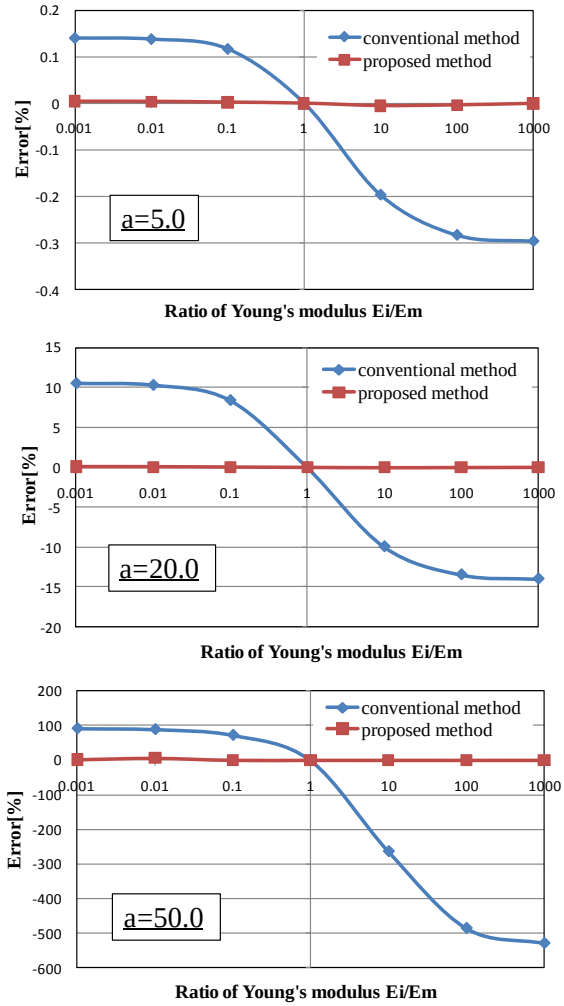


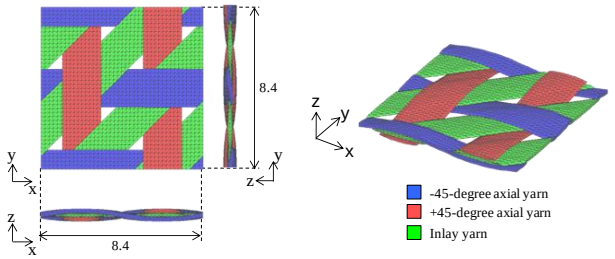
Fig.3 Comparison of numerical accuracy

Fig.3 shows the comparison of the numerical error. The conventional mesh superposition method has large error in case the value of *Ei* and *Em* are not equal. Moreover, as the scale difference between global and local mesh become larger, the numerical error deteriorates. On the other hand, the proposed mesh superposition method is very useful in various *Ei*/*Em* in all scale.

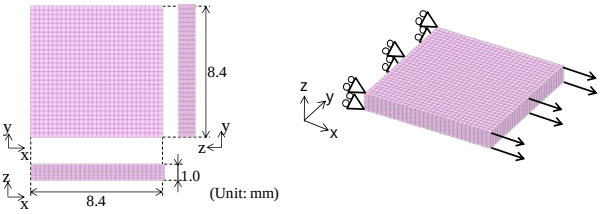
4. Numerical results of mechanical behavior of 2-D braided GFRP

4.1 FE models of 2-D braided composite

As an application of the proposed method to braided composites, the numerical model as shown in Fig.4 is generated. Boundary conditions are applied only to the global model. As comparison of the proposed method, the fine mesh model is also generated with 'WiseTex' program developed by Lomov [4] in Fig.5. Table 1 shows the mechanical properties.



(a) Local model (Fiber bundle part)



(b) Global model and boundary condition

Fig.4 Numerical model with mesh superposition method

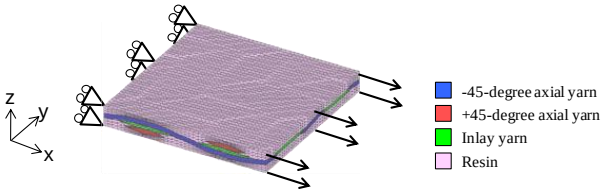


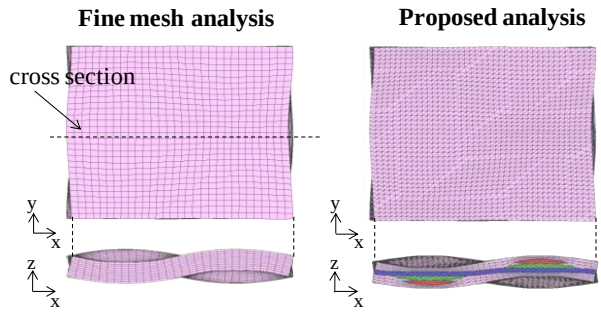
Fig.5 Numerical model with fine mesh

Table 1 Mechanical properties of 2-D braided composite

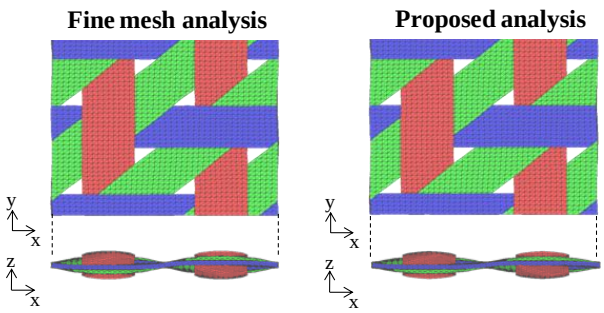
Mechanical properties \ Material	Matrix resin Polyester		Reinforced fiber E-glass		Fiber bundle E-glass/polyester (V_f=0.60)	
Name	Symbol	Value	Symbol	Value	Symbol	Value
Modulus of elasticity [GPa]	E	3.100	E	73.00	E_L	45.04
					E_T,E_Z	12.55
Shear modulus [GPa]	G	1.123	G	30.00	G_{TZ}	4.893
					G_{ZL},G_{LT}	4.688
Poisson's ratio	n	0.380	n	0.217	n_{TZ}	0.282
					n_{LT}	0.067
Tensile strength [MPa]	F	54.00	F	2500	F_L	1501
					F_T,F_Z	87.42
Shear strength [MPa]	F_S	54.00	F_S	–	F_{TZ},F_{ZL},F_{ZL}	97.14
Compressive strength [MPa]	F_C	120.0	F_C	—	F_{CL}	2659
					F_{CT},F_{CZ}	194.3

4.2 Numerical results of 2-D braided composite

Fig.6 shows the numerical results of deformation when strain (ε_x) is 0.1%. Fig.7 shows the numerical results of Z-displacement on the cross section in Fig.6. There are good agreement of Z-displacement with the fine mesh model and the mesh superposition method. In case of the global mesh, the form of undulation transformation out-of-plane direction has also same tendency in both methods.



(a) Resin parts



(b) Fiber bundles parts

Fig.6 Deformation at strain 0.1% (Magnification ratio x200)

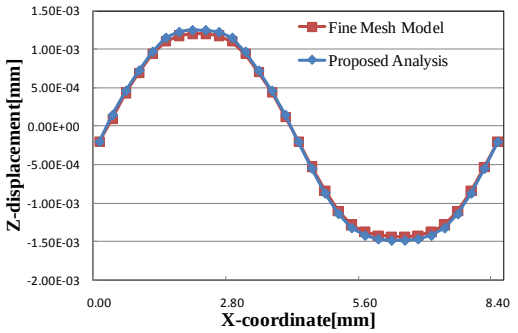


Fig.7 Z-displacement under tensile loading

Fig.8 shows the numerical results of stress distribution when applied strain is 0.1%. The direction L means the material-axis for fiber bundles considering the anisotropy, and the direction T means the perpendicular to the direction L. The tendency of stress distribution has good agreement with both methods.

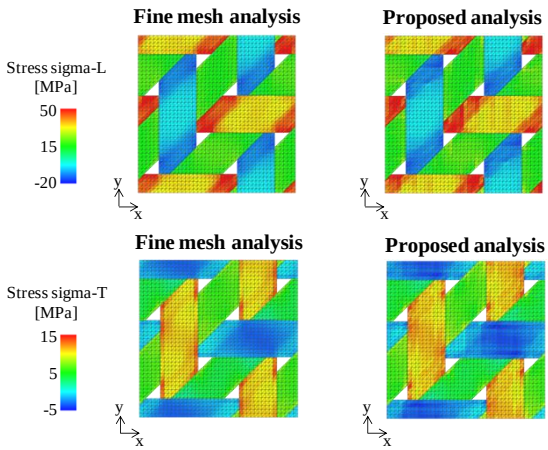


Fig.8 Stress distributions at strain 0.1% (Fiber bundle parts)

The stiffness of the braided composite in X-direction is shown in Table 2. The numerical result with proposed method has a good agreement with the result of fine mesh model.

Table 2 Numerical results of stiffness in X-direction

	Fine mesh analysis	Proposed analysis	Error[%]
Elastic modulus [MPa]	7729	7752	0.30

Fig.9 shows the numerical results of normalized strain energy fraction under the tensile strain 0.1%. There are same tendency of the strain energy fraction in weft, warp, inlay yarn and resin parts with both methods.

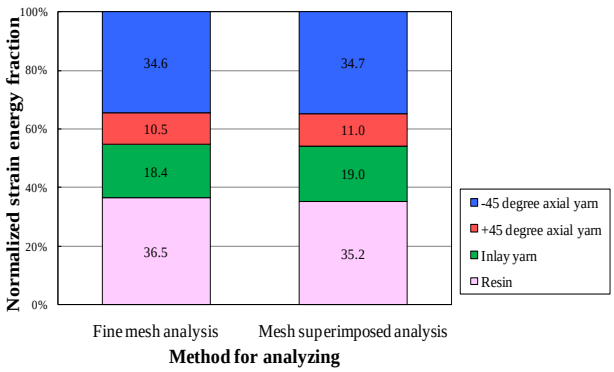


Fig.9 Normalized strain energy fraction

As the results, we believe that the proposed numerical method is very useful to estimate the mechanical behavior of braided composites.

5. Numerical results of mechanical behavior of 3-D braided GFRP

5.1 FE models of 3-D braided composite

Fig.10 shows the 3-D braided composite. Fiber bundles model is generated by adding some process to 2-D braided composite generated by 'WiseTex' program. There are two levels of inlay yarn in the thickness direction, and the axial yarns astride these and intersects.

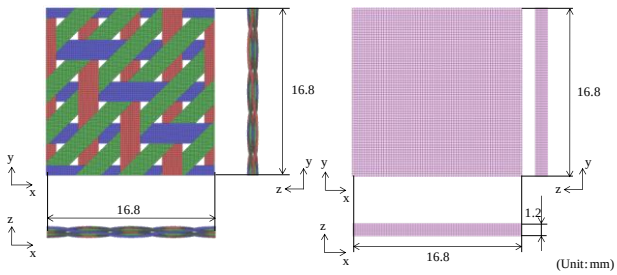


Fig.10 FE model of 3-D braided composites

In order to estimate the difference of the mechanical characteristics, the 2-D braided composite is also prepared as mentioned before. Fig.11 shows the overview of mesh superposition method for 3-D braided composite. Boundary conditions are applied only to the global model. For 2-D braided composite, same analysis is performed. The mechanical properties are used in Table 1.

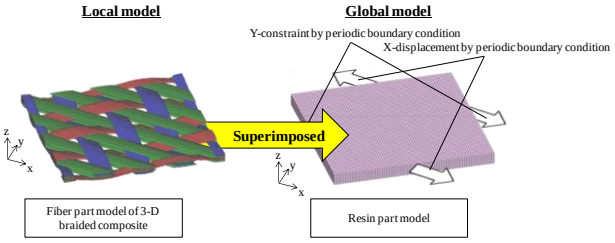


Fig.11 Mesh superposition model of 3-D braided composite

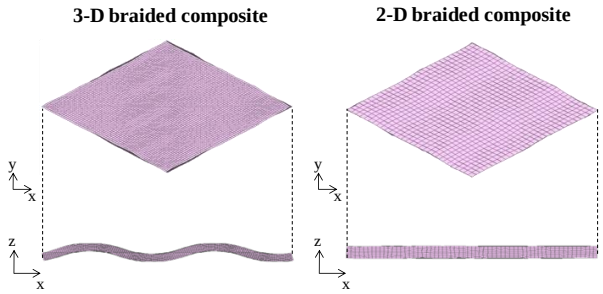
Table 3 shows volume fraction of 2-D/3-D braided composites. There is no layer interval in 3-D braided composite, therefore the volume ratio of the resin part is lower than 2-D braided composite.

Table 3 Volume fraction of 2-D/3-D braided composites

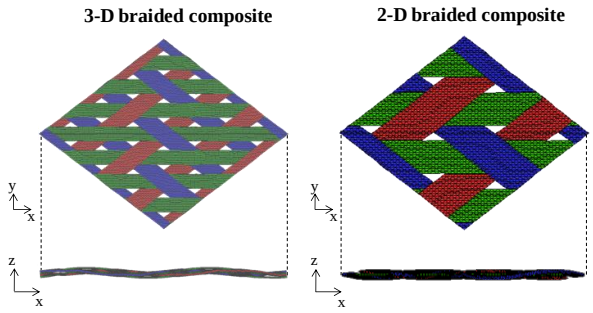
	Volume fraction (%)			
	+45 axial yarn	-45 axial yarn	Inlay yarn	Resin
2-D braided composite	7.3	7.3	10.0	75.5
3-D braided composite	6.1	6.1	14.1	73.8

5.2 Numerical results of 3-D braided composite

Fig.12 shows the numerical results of deformation when applied strain is 0.1%.



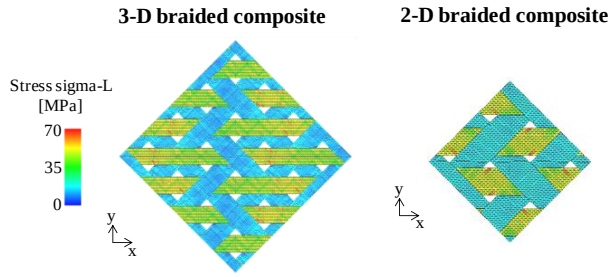
(a) Resin parts



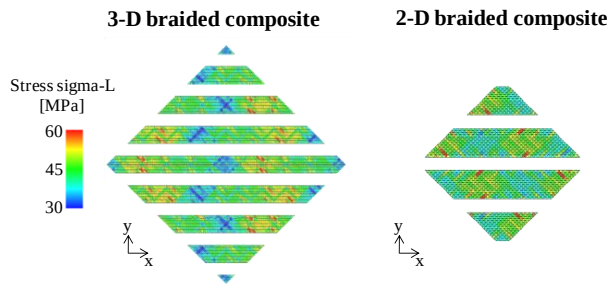
(b) Fiber bundles parts

Fig.12 Deformation under tensile loading in inlay yarn direction at strain 0.1%
(Magnification of deformation: x200)

In 2-D braided composite, the out-of-plane deformation is microscopic because 45-degree and -45-degree axial yarns offset mutual effect. On the other hand, in 3-D braided composite, out-of-plane undulation occurs so that the phase of the undulation of the fiber bundles is different even though 45-degree and -45-degree axial yarns offset mutual effect. Fig.13 shows the numerical results of stress distribution in inlay yarn direction when strain is 0.1%. In the both models, there are almost same tendency of distribution and the stress value. However, in the case of 3-D braided composites, the stress concentration has appeared clearly as comparison with 2-D braided composites, because the 3-D braided composites have the intersectional parts between fiber bundles where the out-of-plane deformation of axial yarns has appeared due to the inlay yarn located in the outside.



(a) Fiber bundles parts



(b) Inlay yarn

Fig.13 Stress sigma-L distribution of fiber part under tensile loading in inlay yarn direction at strain 0.1%

The equivalent material properties of in-lay yarn direction for 2-D/3-D braided composites in inlay yarn direction are shown in Table 4. The modules of 3-D braided composite are higher than that of 2-D braided composite.

Table 4 Equivalent material properties of 2-D/3-D braided composites in inlay yarn direction at non-damaged state

	2-D braided composite	3-D braided composite
Modulus of inlay yarn direction [MPa]	6169	9997

6. CONCLUSIONS

1) We proposed a new mesh superposition method by using the tetrahedral elements as fiber bundle part and the hexahedral elements as resin part.
2) As numerical results of the proposed method, it is confirmed that the proposed method is very useful regardless of the difference of the scale and mechanical properties between global mesh and local mesh.
3) As numerical results of the proposed method for the 2-D braided composite, it is confirmed that the proposed method has a good agreement of the distribution of deformation and stress in fine mesh analysis in any cases.
4) As numerical results of the proposed method for the 3-D braided composite, the mechanical characteristics of 2-D/3-D braided is quite different because of the difference of structure of yarns. From these results, it is revealed that the proposed mesh superposition method with FE mesh of tetrahedral and hexahedral elements is very useful for the estimation of mechanical behavior for braded composites.

References

[1] J.Fish, “The s-version of the finite element method”. *Computers and Structures*, 43(3):539–547, 1992.

[2] T.Kurashiki, M.Zako, et al, “Mechanical Behaviors of Non-Crimp Fabric Composites Based on Multi-scale Analysis”, *Proc. of 17th IC*CM, 2009.

[3] M.Zako, T.Kurashiki, et al, "A multi-scale analysis for structural design of fibrous composites - M^3 method - ", *Proc. of 15th ICCM*, 2005.

[4] S.V. Lomov, E.Bernal, D.S. Ivanov, S.V. Kondratiev, I. Verpoest, “Homogenisation of a sheared unit cell of textile composites: FEA and approximate inclusion model”, *Revue Europeenne de Mecanique Numerique* vol.14 709-728, 2005.