WORK OF SEPARATION OF TRUSS-LIKE MIXED MODE COHESIVE LAWS

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1 Introduction

The concept of cohesive laws, in which the fracture process zone of a material is described in terms of a traction-separation relationship, was introduced in 1960s by Dugdale [1] and Barenblatt [2]. Since Needleman [3] in 1987 implemented a mode I cohesive element in a finite element model, cohesive laws have been widely used in numerical models of materials and structures [4]. Several types of mixed mode traction-separation laws have been proposed. Mixed mode cohesive laws, they can be categorised in three classes: a) uncoupled mixed mode cohesive laws [5], b) coupled mixed mode cohesive laws based on a potential function [6] and c) other mixed mode cohesive laws [7].

Fracture is often observed in layered structures that possess weak fracture planes and often occurs in mixed mode. The fracture process zone will transmit both normal and shear tractions between the crack faces. Experimental studies have shown that the mixed mode fracture energy usually it increases with increasing the phase angle of openings, φ [8]. In this work we examine a class of cohesive laws where the traction vector follows the separation vector.

Such behaviour resembles the behaviour of a truss and thus these cohesive laws are termed truss-like mixed mode cohesive laws. Truss-like mixed mode cohesive laws are attractive for mixed mode fracture problems since the experimental fracture energy as a function of the phase angle of openings can specified and used in the finite element calculations. Apart from the fracture energy for different phase angle of openings, the mode I and mode II cohesive laws are required as inputs.

The purpose is to clarify the conditions under which the work of the cohesive traction (fracture energy) of truss-like mixed mode cohesive laws is independent of the opening path, i.e. when they are derivable from a potential function. The implication of using cohesive laws derived from a potential function is that for a given phase angle of openings, the same work of separation will be attained irrespective of the opening path (normal/shear) history, i.e. identical to the fracture energy specified as input for that phase angle of opening. If not, the work of separation will be different for different paths, although the phase angle of openings is the same.

2 General description of mixed mode cohesive laws

The problem taken up is a planar (two dimensional) cohesive zone problem illustrated in Fig. 1. The entire fracture process zone can be described by a mixed-mode cohesive law.

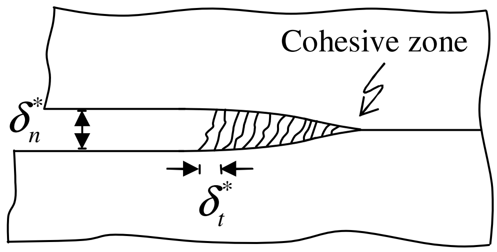

Fig.1. Schematic illustration of a cohesive zone under mixed mode crack opening.

Starting with the path-independent J-integral [9] locally around the fracture process zone, the J-integral becomes:

$$J_R = \int_0^{\delta_n^*} \sigma_n(\delta_n, \delta_t) d\delta_n + \int_0^{\delta_t^*} \sigma_t(\delta_n, \delta_t) d\delta_t \qquad (1)$$

where σ_n and σ_t are the normal and shear tractions, δ_n and δ_t the normal and tangential crack opening displacements, δ^*_n and δ^*_t the normal and tangential crack opening displacements at the end of the cohesive zone as indicated in Fig. 1.

The *J* integral result (Eq. 1) can be interpreted as the work (per unit fracture area) of the cohesive tractions at the end of the cohesive zone. This holds for any values of δ^*_n and δ^*_t. J_R defined according to Eq. 1 is called the fracture resistance. When the cohesive zone is fully developed $\delta^*_n=\delta^f_n$ and $\delta^*_t=\delta^f_t$), J_R equals the work of separation, also called the fracture energy. δ^f_n and δ^f_t are the critical normal and tangential openings for complete failure (the corresponding tractions are equal to zero).

If it is assumed that the tractions are derived from a potential function, Φ, then the normal σ_n, and shear σ_t tractions can be taken to be functions of both δ_n and δ_t but independent of position within the cohesive zone:

$$\sigma_n(\delta_n,\delta_t)=\frac{\partial\Phi(\delta_n,\delta_t)}{\partial\delta_n} \qquad (2)$$

$$\sigma_t(\delta_n,\delta_t)=\frac{\partial\Phi(\delta_n,\delta_t)}{\partial\delta_t} \qquad (3)$$

From Eqs. 1, 2 and 2, the J integral becomes:

$$J_R=\Phi(\delta^*_n,\delta^*_t) \qquad (4)$$

Finally, from Eqs. 2, 3 and 4, the following expressions for the cohesive tractions at the end of the cohesive zone can be obtained:

$$\sigma^*_n=\sigma_n(\delta^*_n,\delta^*_t)=\frac{\partial J_R(\delta^*_n,\delta^*_t)}{\partial\delta^*_n} \qquad (5)$$

$$\sigma^*_t=\sigma_t(\delta^*_n,\delta^*_t)=\frac{\partial J_R(\delta^*_n,\delta^*_t)}{\partial\delta^*_t} \qquad (6)$$

In Eqs 5 and 6 an asterix indicates the position of the end of the cohesive zone. However, since the cohesive laws are assumed to be the same at any position within the cohesive zone, the cohesive laws at the end-openings (Eqs. 5 and 6) must be identical to the cohesive law at any position within the cohesive zone.

The end-openings δ^*_n and δ^*_t (Cartesian form) can be transformed to polar form:

$$\delta^*_m=\sqrt{\delta^{*2}_n+\delta^{*2}_t},\ \varphi^*=\tan^{-1}\left(\frac{\delta^*_t}{\delta^*_n}\right) \qquad (7)$$

where δ^*_m is the end-opening magnitude and φ^* its phase angle of openings. Then the cohesive tractions (Eqs, 5 and 6) can be written as [10]:

$$\sigma^*_n=\cos\varphi^*\frac{\partial J_R}{\partial\delta^*_m}-\frac{\sin\varphi^*}{\delta^*_m}\frac{\partial J_R}{\partial\varphi^*} \qquad (8)$$

$$\sigma^*_t=\sin\varphi^*\frac{\partial J_R}{\partial\delta^*_m}-\frac{\cos\varphi^*}{\delta^*_m}\frac{\partial J_R}{\partial\varphi^*} \qquad (9)$$

3 Truss-like cohesive laws

As mentioned in the Introduction, for truss-like cohesive laws the phase angle of the cohesive traction vector, ψ, and the phase angle of the openings, φ, must be identical for any point within the cohesive zone, $\psi=\varphi$. This also holds for the end-openings. Thus, the direction of cohesive tractions at the end of the cohesive zone must follow the direction of the end-openings:

$$\psi^*=\varphi^* \qquad (10)$$

where the phase angle of the traction vector at the end of the cohesive zone is:

$$\psi^*=\tan^{-1}\left(\frac{\sigma^*_t}{\sigma^*_n}\right) \qquad (11)$$

Then, by substituting Eqs. 8 and 9 into Eq. 11 it can be shown that [10]:

$$\frac{\partial J_R}{\partial\varphi^*}=0 \qquad (12)$$

This implies that when $\psi^*=\varphi^*$ (truss-like cohesive laws) the tractions can be derived from a potential function only when J_R is independent of the phase angle of the openings. Note that it is the fracture resistance, J_R, defined from Eq. 1, not just the fracture energy (the work of separation), that must be independent of φ^* for the tractions to be derivable from a potential function.

4 Truss-like mixed mode cohesive laws with linear softening

A type of widely used truss-like mixed mode bi-linear cohesive laws is used to verify the general result of Eq. 12. Fig. 2 shows a sketch of the bilinear cohesive laws for pure mode I and pure mode II used here. The mode I peak stress is $\hat{\sigma}_n$ and the mode II peak stress is $\hat{\sigma}_t$. The corresponding openings are δ_n^o and δ_t^o.

In the linear softening part the tractions are given by:

$$\sigma_n(\delta_n) = (1-D)K_n\delta_n \tag{13}$$

$$\sigma_t(\delta_t) = (1-D)K_t\delta_t \tag{14}$$

It can be shown that when $\psi=\varphi$, the damage variable D is [10]:

$$D = \frac{\delta_m^f(\delta_n - \delta_m^o\cos\varphi)}{(\delta_m^f - \delta_m^o)\delta_t} = \frac{\delta_m^f(\delta_t - \delta_m^o\sin\varphi)}{(\delta_m^f - \delta_m^o)\delta_t} \tag{15}$$

where δ^o_m and δ^f_m are the mixed-mode critical opening for crack initiation and opening at complete failure, respectively.

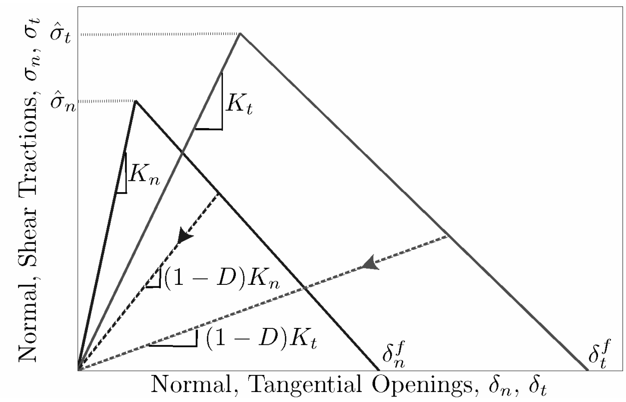

Fig.2. Schematic illustration of bilinear traction-separation laws (normal and tangential directions).

If the cohesive tractions for truss-like mixed mode cohesive laws with linear softening can be derived from a potential function, they must fulfill the following condition [11]:

$$\frac{\partial\sigma_n}{\partial\delta_t} = \frac{\partial\sigma_t}{\partial\delta_n} \tag{16}$$

It can be shown that when the cohesive tractions are described by Eqs. 13 and 14, the criterion given in Eq. 16 is satisfied only when the fracture resistance, J_R, is independent of the phase angle of openings.

5 Numerical verification

The analytical results presented above are verified numerically by the commercial finite element code Abaqus. In order to check the path dependence of the truss-like bi-linear cohesive laws described in Section 4, three opening paths are chosen to the same normal and tangential separations as shown in Fig. 3. If the mixed mode cohesive law is path independent, the work of cohesive tractions should be the same for the three different opening paths.

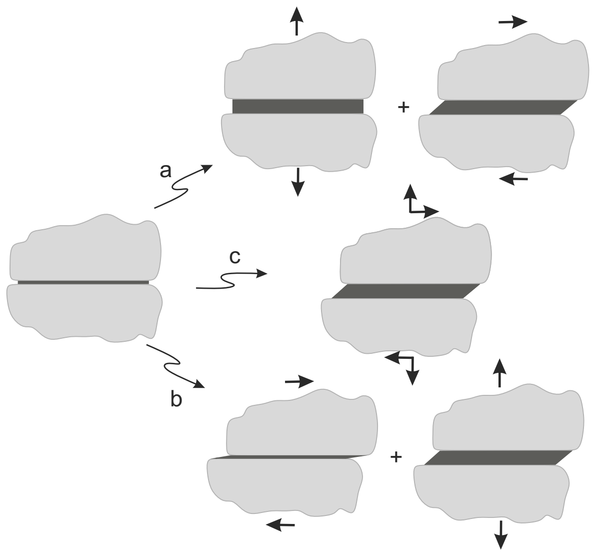

Fig.3. Different loading paths (displacement controlled): a) Normal opening, followed by tangential opening, $\Gamma_{n\to t}$, b) Tangential opening followed by normal opening, $\Gamma_{t\to n}$, and c) proportional loading, Γ_p.

5.1 Method

The two dimensional finite element model consists of a single cohesive element in between two solid elements. Appropriate displacement boundary conditions are applied at the solid elements to result in the opening paths shown in Fig. 3.

Since all nodes of the cohesive element undergo the same normal and tangential separations, the work (per unit area) of the cohesive tractions of the finite element model is calculated as the sum of the mode I and mode II work of cohesive traction, respectively:

$$W_{path} = \int_0^{\delta_n^f} \sigma_n d\delta_n + \int_0^{\delta_t^f} \sigma_t d\delta_t \qquad (15)$$

where δ^f_n and δ^f_t are the normal and tangential openings at which the normal and shear tractions reduce to zero, respectively.

5.2 Results

The work of the cohesive traction along the three different paths is given in Fig. 4 for various ratios of mode II to mode I cohesive fracture energies.

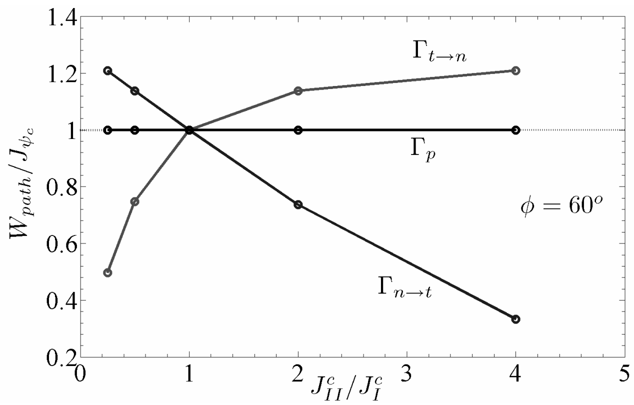

Fig.4. Normalised fracture energy, W_{path}, along paths Γ_p, $\Gamma_{n\to t}$ and $\Gamma_{t\to n}$ for φ=60°. The peak traction value in mode I equals the peak traction value in mode II. J_I^c and J_{II}^c are the fracture energy for pure mode I and pure mode II, respectively.

It is clearly seen that the work of the cohesive traction of the truss-like mixed mode cohesive laws is path independent only when the mode I fracture energy equals the mode II fracture energy. This result confirms the proof of Section 3, and the finding of Section 4. For the results of Fig. 4, it was assumed the mixed mode fracture energy varies linearly between the mode I and mode II fracture energies.

6 Conclusions

A general theoretical proof was given to show that truss-like mixed mode cohesive laws (cohesive laws for which the phase angle of tractions equals the phase angle of the openings) are inherently path dependent except the limiting case where the fracture resistance (and thus the mixed mode traction-separation laws) is independent of the phase angle of the openings. A specific bi-linear truss-like cohesive law, coupled through a failure criterion and an effective displacement, was selected to verify the theoretical analysis. It was shown analytically and numerically that the bi-linear truss-like mixed mode cohesive law is path dependent in accordance with the proof.

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