A NEW FATIGUE LIFE PREDICTION MODEL FOR NOTCHED CFRP LAMINATES

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1 Introduction

In designing reliable structural components made of carbon fiber-reinforced plastic (CFRP) laminates, it is required that the fatigue lives of notched CFRP laminates can efficiently be evaluated for any notch size and for any stacking configuration.

Structural components in engineering applications often contain a notch or hole. Because of stress concentration, local failure in machines and structures during service is apt to initiate at the root or in the vicinity of the notch or hole involved. The degree of reduction in strength of a CFRP laminate due to a notch is governed by the intensity of stress concentration in the vicinity of the notch that is affected not only by the inherent elastic anisotropy but also by the multiple non self-similar cracking in the laminate. It is thus suggested that a CFRP laminate may exhibit notch sensitivity in the range between the notch sensitive and insensitive bounds, depending on stacking configuration and on damage in the vicinity of notch tips.

A more important suggestion in the present context is that the notch sensitivity of a CFRP laminate may not be constant but it may be changed during service by inelastic events occurring at notch tips that are accompanied by the release of local strain energy. In fact, there are many experimental evidences that support the speculation of the variable notch sensitivity in composites. For example, it has been observed that the residual notched strengths of the composites subjected to fatigue loading often become higher than their initial notched strengths [1]. It has also been observed that the S-N relationship for notched specimens of a composite approaches the S-N relationship for unnotched specimens with increasing number of fatigue cycles [2]. These experimental observations not only reveal that the fatigue behavior of notched CFRP laminates is quite different from that of notched metallic materials, but also suggest that the notched fatigue strength prediction method developed for metallic materials is not applicable to CFRP laminates in general.

The present study aims to develop a new engineering fatigue model that explicitly takes into account the change in notch sensitivity due to fatigue loading and facilitates fatigue life analysis of notched CFRP laminates for any notch size. It is an extension of the vanishing notch sensitivity model for fatigue that has recently be developed by one of the present authors. The accuracy of prediction of S-N relationships for notched composites using the proposed notched fatigue model is evaluated by comparing with experimental results.

2 Material and Testing Method

The material used in this study is a twelve-layer woven fabric carbon/epoxy laminate. The nominal thickness of autoclaved laminates after curing was 3.0 mm. The standard (long) specimens with gauge length L_G = 100 mm and width W = 20 mm were used for static tension tests, while the shorter specimens with gauge length L_G = 10 mm and width W = 10 mm were used for static compression tests to reduce the risk of buckling.

Constant amplitude fatigue tests were carried out under load control at room temperature. Fatigue load was applied to specimens in a sinusoidal waveform with a constant frequency of 10 Hz. Most specimens were fatigue tested for up to 10^6 cycles. Each specimen was clamped by the hydraulic wedge grips fitted on the testing machine.

3. Experimental Results and Discussions

3.1 Static Strength

3.1.1 Unnotched and Notched strengths

The unnotched and notched strengths of composite laminates are basic properties, and they are necessary to quantify their unnotched and notched fatigue behaviors. In this study, the notched strength is defined as the maximum nominal stress in the unnotched section [3]; i.e.

$$\sigma_N = \frac{P_{\max}}{A_0} \tag{1}$$

where $P_{\max}$ is the maximum load, and A_0 is the unnotched cross-sectional area of specimens.

The notched tensile and compressive strengths σ_N of the woven CFRP laminate for different hole diameters are compared in Fig. 1, together with the unnotched strength σ_0. The unnotched and notched strengths are averages of two samples, respectively.

When comparing the notched strengths of the woven CFRP laminate either in tension or in compression, we can clearly observe the effect of notch size. The notched strength decreases with increasing hole diameter, in line with the notch size effect observed in other materials. For example, the reduction in tensile strength due to a hole in terms of the ratio of notched to unnotched strength, i.e. notched strength ratio σ_N / σ_0, was 33%, 45% and 61% for different values of diameter-to-width ratio D/W = 0.1, 0.2 and 0.4, respectively.

3.1.2 Notch Sensitivity

The effect of hole size on notched strength can more clearly be observed in the plot of notch strength ratio σ_N / σ_0 against normalized hole diameter D/W. Fig. 2 shows the plots of notch strength ratio σ_N / σ_0 against normalized hole diameter D/W, quantifying the notch sensitivities in the tensile and compressive strengths of the woven CFRP laminate. The dashed lines in this figure indicate the ideally notch insensitive and sensitive bounds. In this figure, it is seen that the normalized notched strength data are distributed in the middle between the notch sensitive and notch insensitive lines, elucidating that the woven CFRP laminate exhibits a moderate notch size effect in the environment tested in this study.

It is seen that the notched strength in tension is slightly larger than that in compression, regardless of notch size. The normalized notched strength data

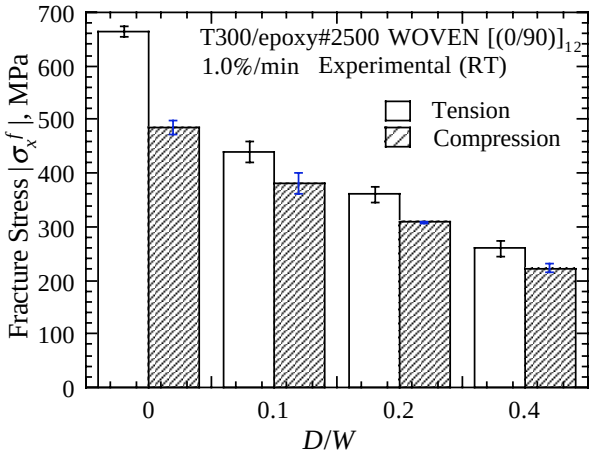


Fig.1. Comparison of notched strengths in tension and compression

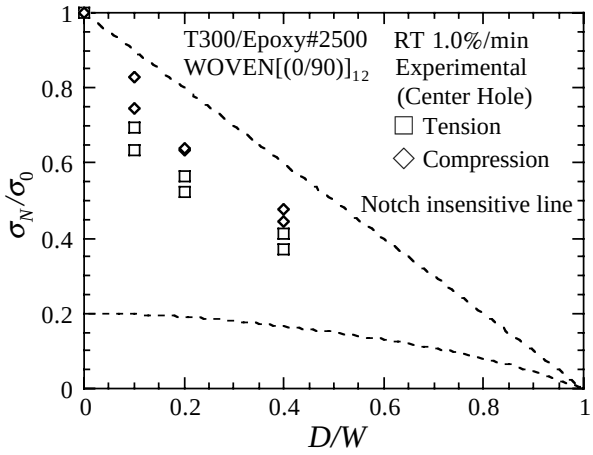


Fig.2. Comparison of notch sensitivities

in compression are distributed more closely to the notch insensitive (NI) line than those in tension, suggesting that the woven CFRP laminate tested in this study is more sensitive to a notch in tension than in compression.

The difference in notch sensitivity between tension and compression is related to the difference in the associated failure modes. Under tension, specimens failed in the notched section almost perpendicular to loading direction. By contrast, specimens failed under compression in an out-of-plane shear mode. The area of fracture surface of a specimen is larger in general when it fails in an out-of-plane shear mode, suggesting that more energy is dissipated in compression by the associated out-of-plane shear failure. This also suggests that notched compressive

failure is relatively more ductile than notched tensile failure. The interpretation of the difference between the notched failure modes in tension and compression is consistent with the lower notch sensitivity observed for compression.

3.2 Unnotched and Notched Fatigue Strengths

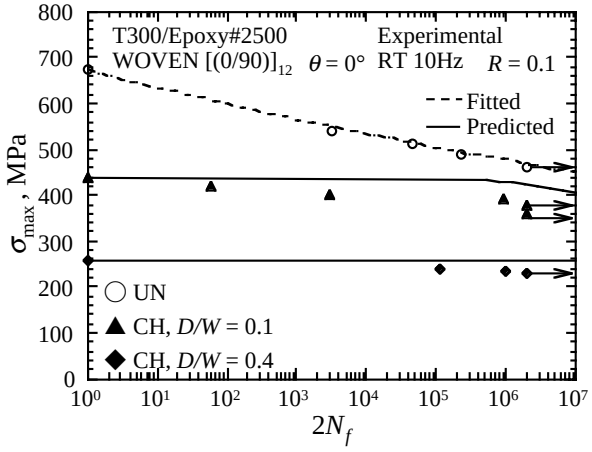
Figs. 3 (a)-(c) show the S-N relationships for notched specimens fatigue loaded at different stress ratios, respectively. These figures include the fatigue data for different hole diameters D/W = 0.1 and 0.4, along with the fatigue data for unnotched specimens D/W = 0 to which a dashed line is fitted. The arrows in these figures indicate the fatigue data run out at 10^6 cycles. In these S-N relationships, the nominal stress in the unnotched gross cross-section is used to describe the maximum level of fatigue stress for notched specimens.

From Fig. 3, it is seen that the S-N relationship for notched specimens is shifted downward with increasing diameter of a hole, because a larger reduction in notched strength occurs with a larger notch. In the case R = 0.1 (Fig.3(a)), the S-N relationship for notched specimens with D/W = 0.1 is almost flat until $N_f = 10^5$ cycles, and it shows a tendency to rapidly approach the S-N relationship for unnotched specimens and then to run almost in parallel with the latter. A similar feature can be seen in the S-N relationship for notched specimens with D/W = 0.1 under fatigue loading at R = –0.5 (Fig.3(b)). These observations suggest that the effect of a notch on the fatigue behavior tends to be reduced during tension-dominated fatigue loading. For specimens with a larger hole, however, this feature cannot be confirmed, since their fatigue lives are beyond the range of observation in this study.

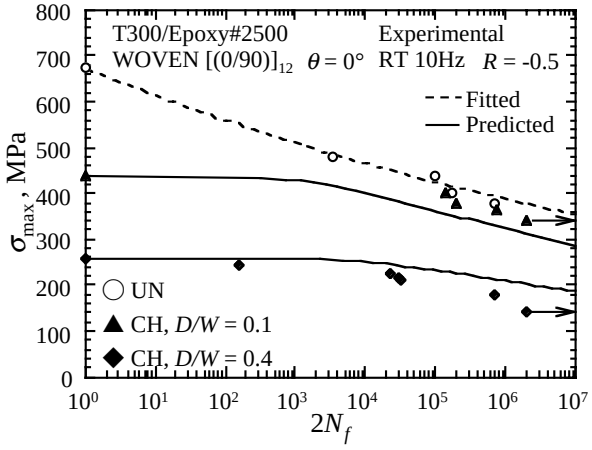
In the case of R = 10 (Fig.3(c)), no appreciable tendency for the S-N relationships for notched specimens to rapidly approach the S-N relationship for unnotched specimens can be seen. Rather, the former runs almost in parallel with the latter over the whole range of fatigue life tested in this study. This observation suggests that the composite tested in this study remains notch sensitive under compressive fatigue loading.

4. Unnotched Fatigue Strength Model

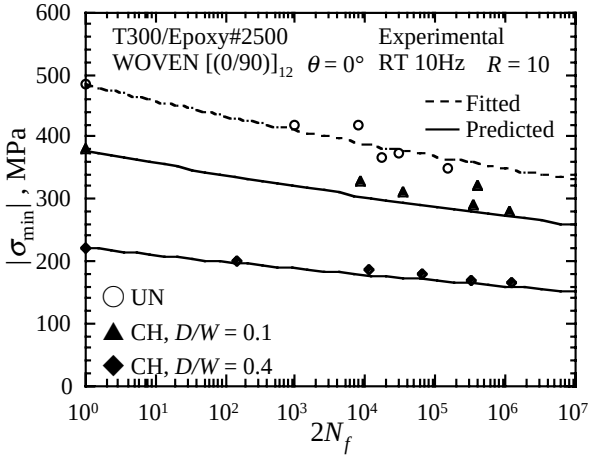
In the previous study [4], it was assumed that the intensity of fatigue damage in orthotropic



(a) R = 0.1



(b) R = –0.5



(c) R = 10

Fig.3. Comparison of the S-N relationships for unnotched and notched specimens

composites can be described using a single scalar variable ω , and its evolution equation can be prescribed by the following equation:

$$\frac{d\omega}{dN} = K\psi^n \left(\frac{1}{1-\omega} \right)^k \frac{\langle \psi - \psi_L \rangle^b}{\langle 1-\psi \rangle^a} \tag{2}$$

where ψ is the fatigue strength ratio defined as

$$\psi = \frac{\sigma_{\max}}{\sigma_0} \tag{3}$$

The coefficients K, k, n, a, b and ψ_L are fatigue constants; while ψ_L corresponds to a normalized fatigue limit in case of appearance. The angular brackets $\langle \ \rangle$ denote the singular function defined as $\langle x \rangle = \max\{0, x\}$.

Integration of Eq. (2) with the assumption that fatigue failure occurs at $\omega = 1$ yields the following form of fatigue life equation for unnotched fatigue behavior:

$$N_f = \frac{1}{(1+k)K\psi^n} \frac{\langle 1-\psi \rangle^a}{\langle \psi - \psi_L \rangle^b} \tag{4}$$

The fatigue life equation given by Eq. (4) includes three basic non-dimensional functions:

$$\Phi_0^S = \langle 1-\psi \rangle^a \tag{5}$$

$$\Phi_0^L = \langle \psi - \psi_L \rangle^b \tag{6}$$

$$\Phi_0^F = \psi^n \tag{7}$$

These basic functions play the following important roles. The function Φ_0^S allows to describe the fatigue behavior in which fatigue life rapidly shortens as maximum fatigue stress approaches static strength; i.e. the description of static strength dependence of fatigue life. The function Φ_0^L furnishes the fatigue life equation with a capability to predict infinite life for maximum fatigue stress below a fatigue limit. The use of the function Φ_0^F suggests that the S-N relationship for a middle level of maximum fatigue stress between the static strength and a fatigue limit is approximately described by means of a power-law function of the Basquin type [5].

5 Notched Fatigue Strength Model

The fatigue life equation for unnotched composites is modified into a form that can be used for predicting the S-N relationships for notched composites under a given constant amplitude fatigue loading.

First, we replace the function $\Phi_0^{(S)}$ for unnotched composites with that for notched composites defined as

$$\Phi_N^{(S)} = \langle 1-\psi_N \rangle^a \tag{8}$$

Next, it is assumed that in the case of tension dominated fatigue loading the notch sensitivity vanishes in the range of long life and the S-N relationship for a notched composite approach that for an unnotched composite, while in the case of compression dominated fatigue loading it lasts. Accordingly, the basic functions that controls the fatigue behavior in the range from a middle to low stress level are replaced with the following functions:

$$\Phi_N^{(F)} = \left(\psi_N^{(F)} \right)^n \tag{9}$$

$$\Phi_N^{(L)} = \left\langle \psi_N^{(F)} - \psi_{N(L)}^{(F)} \right\rangle^b \tag{10}$$

where $\psi_N^{(F)}$ is a fatigue notch strength ratio that is introduced to distinguish the notch sensitivities for different stress ratios, and it is defined as

$$\psi_N^{(F)} = \begin{cases} \psi_{NI}, & \chi_N \le R < 1 \\ \psi_N, & 1 < R, \ -\infty \le R < \chi_N \end{cases} \tag{11}$$

The parameter $\psi_{N(L)}^{(F)}$ is introduced to deal with the S-N relationship associated with a fatigue limit that depends on stress ratio in general. In case of no influence of a fatigue limit on the S-N relationship, it is simply assumed that $b = 0$.

Replacement of the three basic functions for unnotched composites with their modified versions yields the following fatigue life prediction equation for notched composites:

$$N_f^{(N)} = \frac{1}{\left(1+k\right)K\left(\psi_N^{(F)}\right)^n} \frac{\left\langle 1-\psi_N \right\rangle^a}{\left\langle \psi_N^{(F)} - \psi_{N(L)}^{(F)} \right\rangle^b} \tag{12}$$

The material constants involved by Eq. (12) for notched composites are assumed to be identical with those involved by Eq. (4) for unnotched composites. Thus, they are determined by fitting Eq. (4) to the S-N relationship for unnotched specimens. It is obvious that we can reduce the fatigue life equation, Eq. (12), for notched composites to the fatigue life equation, Eq. (4), for unnotched composites by assuming as $\alpha = 0$.

Once the static strength and notch sensitivity parameter of a composite and the S-N relationship for unnotched specimens are identified, the S-N relationship for notched specimens of the composite can be predicted using the formula given by Eq. (12) for any notch size. It is emphasized that two capabilities to predict the notched static and fatigue strengths of a given composite are furnished in a unified manner with the variable notch sensitivity approach proposed in this study.

Note that the material constants K, k, n, a, b and ψ_L involved by Eq. (2) characterize the S-N relationship for unnotched specimens for a given stress ratio, and they take different values for different stress ratios in general. These material constants are identified by fitting Eq. (2) to the S-N relationship for a given stress ratio. If the fatigue data on unnotched specimens are available for the stress ratios to be considered, they are used to determine the material constants. Even if the experimental S-N relationships for unnotched specimens for the stress ratios imposed by notched fatigue analysis are not available, they can be predicted using the anisomorphic CFL diagram approach proposed in the previous studies [6].

The predictions using the proposed fatigue model for notched composites are indicated in solid lines in Figs. 3 (a)-(c). From these figures, it can be confirmed that the predicted S-N relationships for notched specimens agree well with the experimental results, regardless of notch size and stress ratio. It is emphasized that the notched fatigue data were not used for material identification. Thus, it may be concluded that the variable notch sensitivity approach developed in this study can be used to predict the fatigue lives of notched composite laminates for any notch size over a range of stress ratio.

6. Conclusions

The notched fatigue behavior for a woven fabric carbon/epoxy laminate was examined for different notch sizes and stress ratios at room temperature. A new fatigue life prediction method for notched composites was then developed that can be applied to specimens with a notch of any size and to fatigue loading at any stress ratio. Finally, the validity of the proposed notched fatigue life prediction method was evaluated by comparing with experimental results.

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