THE HOMOGENIZED VISCOELASTIC AND RATE DEPENDENT PLASTIC MODEL FOR PLAIN WEAVE FABRIC REINFORCED POLYMER COMPOSITES

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1 Introduction

Composites reinforced by woven fabric are of increasing interest in many diverse and unique applications such as aerospace, automotive, marine, and military. The woven fabric composites have been found more effective than the unidirectional composites due to its reinforcement in all directions within a single layer. In addition, they also have other advantages including better impact resistance, better toughness, and good conformability to complex shape. These properties make the woven fabric composites attractive in many structural applications. Hence, the analysis of woven fabric composites to characterize their behavior is an area of active interest to researchers. In general, the behavior of composites strongly depends on the geometric and mechanical properties of its constituents, i.e., reinforcing fibers and matrix, and the fabric architecture. Many researchers have investigated this homogenization technique for analyzing behaviors of whole composite. Ishikawa and Chou [1] developed three analytical one-dimensional models, known as mosaic, crimp, and bridging models. These models consider only the undulation of the yarns in the loading direction by using classical laminate theory. Naik and Shembekar [2] and Naik and Ganesh [3] extended the above model to two dimensional elastic models. They used parallel-series assumptions for their micro-mechanical models based on classical laminate theory. Naik [4] developed three-dimensional elastic models of woven and braided fabric reinforced composites. These models were incorporated in computer code for failure analysis of fabric composites. The yarns are divided in many slices and these slices are homogenized assuming sinusoidal undulation of yarns. The homogenization of the material properties is based on iso-strain assumption. Karayaka and Kurath [5] proposed micro-mechanical model based on mixed boundary conditions, which are iso-strain and iso-stress homogenization technique. Tabiei and Jiang [6] developed a micro-mechanical model with non-linear constituent. In this model, the representative volume cell (RVC) was divided into many sub-cells and the equivalent properties were obtained by an averaging technique.

Characterization of strain rate dependent behavior of composites is essential to predict the dynamic behavior such as impact or creep phenomena associated with them. High strain rate experiments have been performed on epoxy in unidirectional compression only to determine a rheological model for the epoxy [7]. A spring in parallel with two Maxwell elements is used to get the time-dependent constitutive for the visco-elastic material. Wang [8] showed that two Maxwell elements are enough to get the high strain rate response of epoxy, though the epoxy was defined as nonlinear model. Karim [9] used a two-term linear spring dashpot system to get the high strain rate of the epoxy. In general, the constants of rate dependent behavior are determined from experimental results involving stress and strain curve, strain rate, and the time of load application. A common form for these constituent employs a Prony series.

In this paper, rate dependent mechanical behavior of plain woven carbon fabric reinforced plastic composite is investigated. The polymer matrix is assumed isotropic and visco-elastic in elastic region. The visco-elastic behavior is defined as time domain Prony series. Rate-dependent behavior in plastic region is investigated by Split-Hopkinson Pressure Bar (SHPB) test. The coefficients of Prony series are obtained inductively from the SHPB test result.

2 Rate-dependent constituent

2.1 Homogenization

Rao, Mahajan and Mittal [10] proposed a homogenization technique based on the principles of average stress-average strain as follows.

$$\langle \varepsilon_{ij} \rangle = \frac{1}{|V_e|}\int_V \varepsilon_{ij}^0(x,y)dv_e = \varepsilon_{ij}^{(0)} \tag{1}$$

$$\langle \sigma_{ij} \rangle = \frac{1}{|V_e|}\int_V \sigma_{ij}^0(x,y)dv_e = \left[C_{ijkl} + C_{ijmn}\frac{\partial X_m^{kl}}{\partial y_n} \right]\varepsilon_{kl}^{(0)} = C_{ijkl}^H(\varepsilon^0)\langle \varepsilon_{kl} \rangle \tag{2}$$

where $\varepsilon_{ij}^{(0)}$ is uniform strain, σ_{ij}^0 is uniform stress, ∂X_m^{kl} is a periodic function representing characteristic modes of the unit cell and C_{ijkl}^H is the equivalent homogenized stiffness coefficients. In the present study, we used this technique to investigate the unit model of plain woven consisting 3 layers and of these 2 have 0.25 mm thickness, fabric layers and a single resin layer.

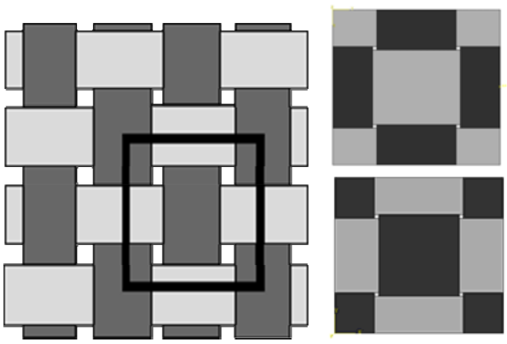

Fig. 1 Representative Unit cell and two fabric layers

To validate this model, a example analysis is performed. The composite is made of Hercules AS4 graphite fibers impregnated with Hercules 3501-6 epoxy matrix. The fiber diameter is 0.007 mm and fiber volume fraction is 50%. The ideal yarn fiber packing density of 0.78 is used in this analysis.

Table 1. Yarn and resin properties

Material	E_{11} (GPa)	E_{22} (GPa)	G_{12} (GPa)	ν_{12}	ν_{23}
Yarn	144.8	11.73	5.52	0.23	0.3
Resin	3.45	3.45	1.28	0.35	0.35

2.2 Quasi-static compression test

Fig. 2 Instron 6567 50kN Universal testing machine

The specimens were manufactured as 48 plies and by vacuum bag molding method. The stacking sequence was $[45/-45/0/90]_{6s}$. They were cured at 350˚F and 41 psi for 4 hours. The specimens were plain weave carbon fabric/epoxy laminates of size 10 × 12 × 6 mm. For the quasi-static test, an universal material test machine, Instron 6567, of capacity 50kN is used, as shown in fig. The compression test is displacement control. And the strain rate is approximately 0.002 /s.

2.3 Resin visco-elasticity

Polymers are generally visco-elastic but some exhibit visco-plasticity, i.e. they undergo permanent deformation upon loading. Most experimental results on fiber-reinforced polymers indicate that regardless to the mode of loading, the modulus increases with increasing strain rate, and the strength and fracture strain may increase depending on the range of strain rate. Since the same rate-dependent behavior had been observed for unreinforced polymeric materials, it is assumed that the rate-dependent characteristics of the composite can be described using time-dependent or viscoelastic property of the polymer. The rate-dependent constitutive equation for carbon/epoxy composite material is exptressed in terms of a time-dependent relaxation modulus. The time-dependent relaxation modulus is found by considering how the material relaxes under constant strain. The constitutive equation is then expressed by linear hereditary law. Since all the high strain rate experimental results available to us are carried out at constant strain rate, the hereditary law based on constant strain rate is used to find material properties appropriate for a high strain rate constitutive law. Thus, the homogenized constituent can be expressed as:

$$\langle \varepsilon_{ij} \rangle = \frac{1}{|V_e|}\int_V \varepsilon_{ij}^0(x,y,t)dv_e = \varepsilon_{ij}^{(0)} + \int_0^t \dot{g}_R(s)\varepsilon(t-s)ds \quad (3)$$

$$\langle \sigma_{ij} \rangle = \frac{1}{|V_e|}\int_V \sigma_{ij}^0(x,y,t)dv_e$$
$$= \left[E_{ijkl} + E_{ijmn}\frac{\partial X_m^{kl}}{\partial y_n}\right]\left[\varepsilon_{kl}^{(0)} + \int_0^t \dot{g}_R(s)\varepsilon(t-s)ds\right]$$
$$= E_{ijkl}^H(\varepsilon^0)\langle \varepsilon_{kl} \rangle \quad (4)$$

We can assume that the viscoelastic material is defined by a Prony series expansion of the dimensionless relaxation modulus;

$$\sigma(t) = E_0\left(\varepsilon - \sum_{i=1}^N \varepsilon_i\right) \quad (5)$$

where, $\varepsilon_i = \frac{\bar{g}_i^P}{\tau_i^G}\int_0^t e^{-s/\sigma_i^G}\varepsilon(t-s)ds$

2.4 Rate-dependent plastic behavior

For high strain rate testing, a modified SHPB apparatus was used. The SHPB apparatus consists of 16 mm diameter Inconel 718 striker, incident and transmitter bars which are 300, 1600 and 1600 mm in length, respectively.

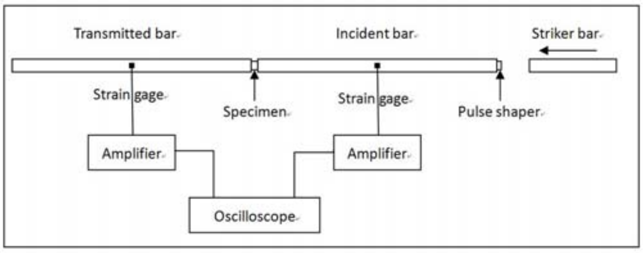

Fig 3. SHPB apparatus

3 Results and Discussion

3.1 Validation of unit-cell model

The elastic behavior of 3 layer unit model is compared with values obtained from mmTEXlam and experiments [11]. The results are compared in table 2. The 3-layer unit model shows similar response to test result.

Table 2 Material homogenization comparison among 3 layer unit-cell model, 1 layer model and test results

Material	E_{xx},E_{yy} (GPa)	G_{xy} (GPa)	ν_{xy}
1 layer (mmTexlam)	64.38	5.64	0.23
3 layer	61.99	3.80	0.21
Test	61.92	-	0.11

3.2 Comparison between experimental and computational result for quasi-static compression test

The compression test was conducted on [45/-45/0/90]$_{6S}$ specimen. The elastic compression modulus of the fabric is calculated as 5.792 GPa from 3-layer FEM analysis. The properties of resin are same as in table 1. Fig. 4 shows the results of experiment and finite element analysis quasi-static compression of [45/-45/0/90]$_{6S}$ CFRP specimen. The static yield stress is observed as 200 MPa at strain 0.08. In the experimental result, the stress value increases after the drop at 200 MPa, which this can be regarded as yield limit and failure. The stress-strain behavior shows very good conformity between experimental and FEM results.

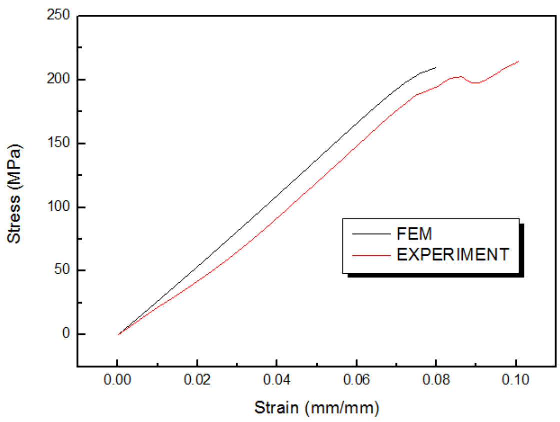

Fig 4. Comparison between Quasi-static compression test and FEM result

3.3 Rate dependent behavior

The plastic behaviors under strain rates of 400/s, 1200/s, and 1500/s are plotted in Fig. 5, obtained from the SHPB test. The plastic properties like flow stress, ultimate strength, and strain hardening can be obtained separately from these graphs. In numerical analysis, each can be expressed as a general empirical plastic constituent called Johnson-Cook plasticity without temperature effect. The yield stress for Johnson-Cook strain-rate dependence is expressed as below.

$$\bar{\sigma} = \left[A + B\left(\bar{\varepsilon}^{pl}\right)^{n} \right] \left[1 + C \ln\left(\frac{\dot{\bar{\varepsilon}}^{pl}}{\dot{\varepsilon}_0} \right) \right] \qquad (6)$$

The left term is for strain hardening and the right presents strain rate hardening behavior. A, B, n and C are material constants are presented in Table 3.

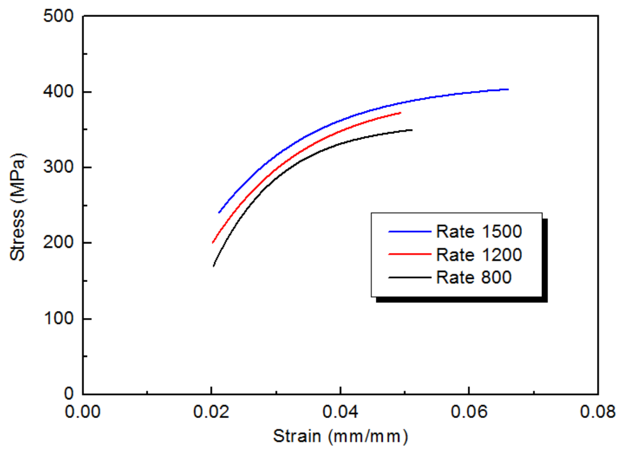

Fig. 5 Stress-Strain curve result from SHPB test

Table 3 The material properties of Johnson-Cook plasticity for the CFRP [45/-45/0/90]$_{6S}$ composites .

Material	A (MPa)	B (MPa)	n	C
CFRP [45/-45/0/90]$_{6S}$	200	450	0.2	5

Though the experimental result shown in Fig.5 do not imply elastic behavior, in composites the estimation of exact flow stress is severely difficult because of short plastic range; and therefore, the Johnson-Cook plasticity is useful to find rate dependent elasticity of composite materials.

3.4 Prediction of Prony coefficients

The flow stress for each strain rate calculated from SHPB test r can be used to define the visco-elastic coefficients of Prony series, $\bar{g}_i^P$ and τ_i^G . The practical compliance range expected from SHPB test are shown in fig. 6. To find Prony constants, expected modulus of strain rates are convert to compliances and each compliance is expressed as ratios about static compliance as shown in table 4.

Table 4 Expected elastic modulus at 800, 1200, and 1500 /s strain rates

Strain rate (/s)	Compliance (/Gpa)	Compliance Ratio (%)
0.0002 (Qusi-static)	0.47	100
800	0.105 ~ 0.118	22.3 ~ 25.1
1200	0.094 ~ 0.100	20.0 ~ 21.3
1500	0.083 ~ 0.087	17.7 ~ 18.5

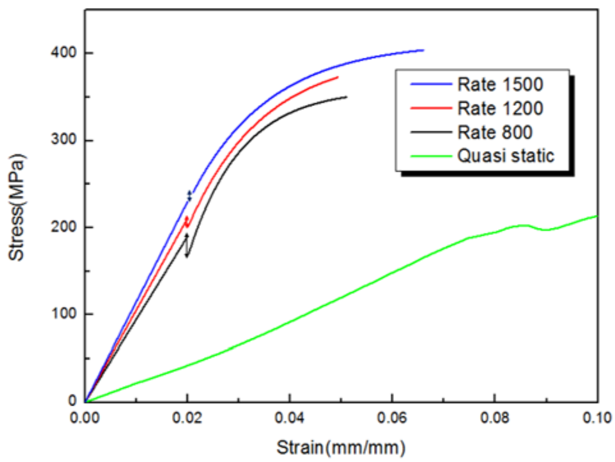

Fig. 6 Expected elastic modulus range

The compliance reduction ratio can be determined from the expected elastic modulus. And Prony constants can be extracted by compliance reduction ratio analysis like table 5.

Table 5 Viscoelastic prony constants for the CFRP [45/-45/0/90]$_{6S}$ composites .

Material	$\bar{g}_i^P$	K_i^G	τ_i^G (s)
CFRP [45/-45/0/90]$_{6S}$	0.8	-	0.000015

Fig. 7 and Fig. 8 show the maximum and the specific time-normalized compliance graphs calculated by Prony constants listed in table 5.

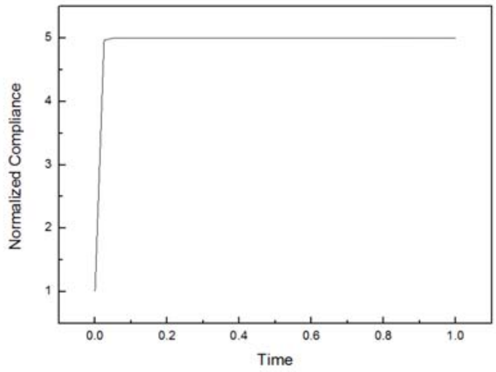

Fig. 7 Compliance increment along time

Fig.7 indicates a very short viscoelastic behavior period with the static compliance converging to 5. The computational analysis is performed by ABAQUS/explicit. Boundary velocity and analysis time for strain rates, 800 /s, 1200 /s and 1500 /s are represented in table 6.

Table 6 Parameters for compliance analysis

Strain rate (/s)	Boundary velocity (mm/s)	Analysis time period(s)
800	8640	2.5E-5
1200	12960	1.67E-5
1500	16200	1.33E-5

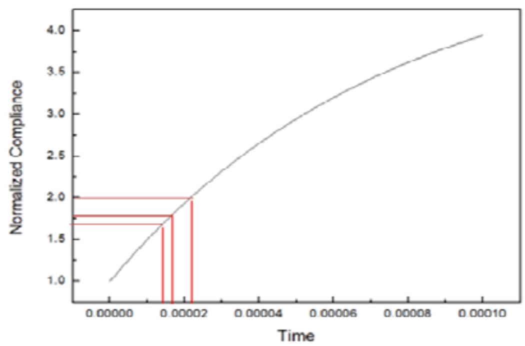

Fig. 8 Compliance increment along time in short time range.

In Fig.8, for strain rate of 800 /s, analysis time is 2.5E-5 and the normalized compliance is 2 at the moment. And, the compliance reduction ratio is 25%. Likewise, at strain rate 1200 /s and 1500 /s compliance reduction ratios are 20% and 18%, respectively.

3.5 Comparison with homogenized numerical model

Fig. 9 compares the computational results for stress-strain behavior of CFRP $[45/-45/0/90]_{6S}$. This model is homogenized by 3 layer unit cell model with Johnson-Cook plasticity. Because of short period dynamic analysis, there are stress retardations in elastic range.

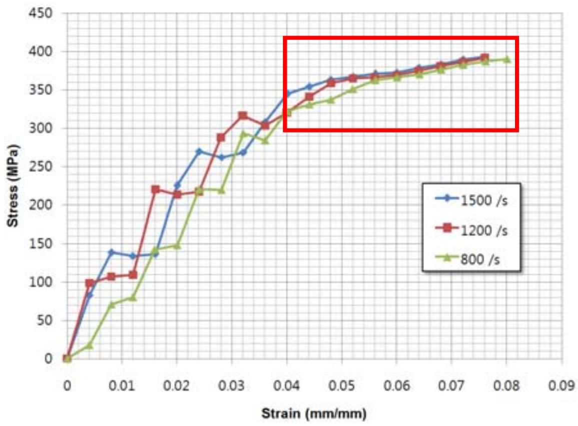

Fig. 9 The rate dependent mechanical behavior of CFRP $[45/-45/0/90]_{6S}$ composite.

SHPB results for different strain rate showed very small deviation in their stress-strain curves. Thus, the rate dependent behavior was not significant. This can also be observed in Fig.10, which is magnified view of computational curves of Fig.9 in red box.

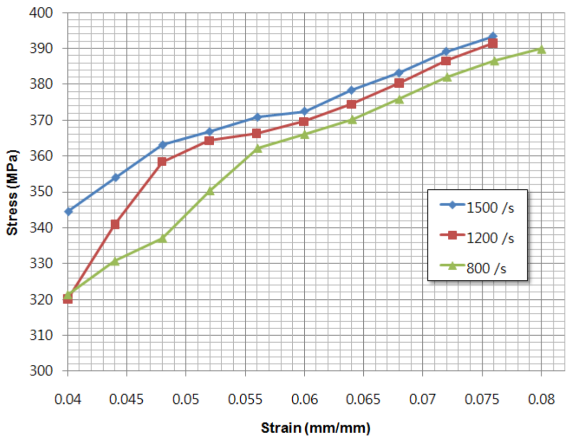

Fig. 10 The rate dependent plasticity of CFRP $[45/-45/0/90]_{6S}$ composite.

Fig. 10 shows Johnson-Cook plastic hardening with strain rate. Because the rate dependent hardening of Johnson-Cook plasticity is logarithm function, the flow stress difference between 800 /s, 1200 /s, 1500 /s is not large compared with that of 10000 /s, 1000 /s, and 1 /s. Nevertheless the numerical results fit well to the experimental result as shown in Fig. 11.

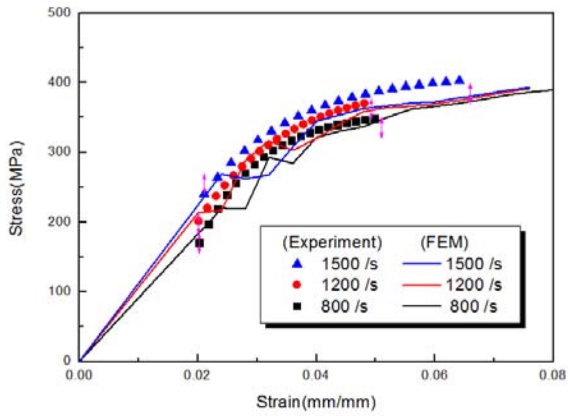

Fig. 11 The superposed total mechanical behavior with viscoelasticity and rate dependent plasticity.

The material model include homogenized visco-elastic and rate-dependent plasticity applied simultaneously.
In general, it is very difficult to measure or even observe plastic behavior in composites, particularly the rate dependent behavior in short time like impact loading. Therefore, from practical point of view the computational analysis and techniques of this study can of significant help to composites and impact engineering fields. In addition, it can be extended to include long term viscoelasticity.

4 Conclusion

Unit cell model of plain woven polymer composite is validated after comparison against commercial software, mmTEXlam. The strain rate dependent material constituent is inclusively expressed in the elastic and plastic regions. This constituent behavior is analyzed for woven fabric polymer composite under dynamic behavior like impact or creep.

Acknowledgement

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