INTEGRATED DESIGN AND PRODUCTION OF FILAMENT-WOUND COMPOSITE STRUCTURES: COMPROMISE BETWEEN STRENGTH AND MANUFACTURABILITY

Lei Zu, Jihui Wang, Shuxin Li

School of Materials Science and Engineering, Wuhan University of Technology, 122 Luoshi Road, Wuhan 430070, China
Email: zulei@whut.edu.cn, web page: http://www.whut.edu.cn

Keywords: Composite pressure vessels, Filament winding, Isotensoid, Geodesic, Non-geodesic

ABSTRACT

One of the most important issues for design and production of filament-wound composite structures reflects on the determination of the most efficient geometries and related fiber architectures. To better understand the integrated design and production of filament-wound composite structures, in this paper we present an overview and comprehensive treatment for a variety of filament-wound composite pressure vessels. The fibers must be stable on the mandrel and be exactly placed along trajectories as predetermined by structural design. To obtain a stable fiber trajectory, the stability-ensuring conditions are formulated in terms of both fiber slippage and bridging tendencies; these conditions give the basic criteria for the subsequent design of filament-wound pressure vessels. A new possibility to improve the structural performance can be offered by applying isotensoid cross-sectional shapes instead of the conventional shapes. The isotensoid design, which leads to equal fiber stress throughout the whole structure, is conducted to determine the netting-based optimal cross-sections. Geodesics and non-geodesics-based isotensoid domes and toroids are respectively outlined as examples for the isotensoid design. The results show that the isotensoid cross-sectional shapes lead to significantly improved structural performance of filament-wound pressure vessels and provide an effective tool able to fill the gap between "design for structures" and "design for manufacturing".

1 INTRODUCTION

In the past few decades filament-wound composite structures have gained a widespread application not only for military but also for civilian uses. Filament winding technique is an effective tool to manufacture rotationally symmetrical composite structures, such as composite pressure vessels and piping [1-4], solid rocker motor case [5-7], and drive shafts [8-10]. In general, a filament-wound composite pressure vessel constitutes an inner liner and a composite overwrap. The liner is primarily used to prevent leaking of the storage substance, while some of the liners, depending on their stiffness and thickness, do also provide considerable strength to carry the internal pressure load. Filament-wound composite pressure vessels should take full advantage of the ultimate tensile strength and elastic modulus of the wound fibers, in order to achieve their optimal structural performance.

The design of fiber trajectories is based on two criteria: optimal lay-up structures [11-15] and suitable winding patterns [16-20]. The fiber trajectories should be able to fulfill design requirements for either strength or manufacturability, e.g. maximum structural performance, maximum fiber strength, uniform stress distribution, non-slippage, non-bridging, uniform and full coverage. A major advantage of composite materials is the large number of design variables available to the designer. To realize this potential and to maximize the structural properties which composites can offer, the design has to properly capture the specific requirements of the problem. Optimal design is an effective way of achieving this goal. A compromise strategy between "design for structures" and "design for manufacturability" must be sought.

The generalized optimality condition originates from the idea that the optimal pressure vessels are
Lei Zu, Jihui Wang and Shuxin Li

governed by the condition of equal shell strains (principal stress design), which results in the complete occupation of the fiber strength and the participating layers aligned in the direction of the maximum principal stress. It has also been proved that the optimal cross-sectional profile for a filament-wound pressure vessel is an isotensoid [21], on the basis of the netting theory [22]. The isotensoid implies that all the fibers undergo uniform tension along their length. The isotensoid-based solution may be regarded as optimal since it guarantees uniform stress distribution, minimum structural weight and maximum occupation of the fiber strength.

In this paper we present an overview and perform an elaboration of the design and optimization procedure associated with filament-wound domed and toroidal pressure vessels. Among possible shapes of pressure vessels, toroids have been recently gaining increasing attention for the storage of pressurized liquids and gases, due to their high structural efficiency and novel configuration. Since the dome regions are the most critical with regard to structure failure, the design of the domes became also one of the most imperative issues in pressure vessel design. Beginning with the derivation of the stability-ensuring conditions for fiber trajectories and the outline of the integrated design procedure, we proceed to the isotensoid design for the domed and toroidal pressure vessels, based on geodesic and non-geodesic trajectories. Non-geodesics are here applied to replace the conventionally used geodesics, in order to improve the structural performance and produce unique vessel shapes, e.g. domes with unequal polar openings. The isotensoid cross-sectional profiles are derived and prove to exhibit better performance than the traditional ones. Finally, some conclusions and recommendations are drawn.

2 STABILITY-ENSURING CONDITIONS FOR FIBER TRAJECTORIES

An analysis for the stability of fiber trajectories is imperative which allows accurate fiber placement to achieve the optimal structural performance dictated by structural design. Any deviation from geodesics will require a lateral force of friction to prevent the fiber sliding from its original trajectory. Furthermore, for a concave surface the condition which avoids fiber bridging should also be respected. The vector parameterization of an arbitrary mandrel surface in generalized curvilinear orthogonal coordinates can be represented as:

\[
S = S(u, v)
\]  
(1)

where \( S \) is a vector-valued function of the parameters \((u, v)\) and the parameters vary within a certain domain in the parametric \( uv \)-plane. An infinitesimal elementary piece \( P_0P_1 \) (\( |P_0P_1| = ds \)) of a fiber curve is here considered, as depicted in Fig. 1. \( P_0P_1 \) is subjected to longitudinal tension forces \( F_0 \) and \( F_1 \), a normal reaction force \( F_n \) perpendicular to the surface and a friction force \( F_f \) tangential to the mandrel surface. The Darboux frame [23], which takes into account the fact that the curve \( C(s) \) lies on the surface \( S(u, v) \), is here considered. Using the right-handed orthonormal basis \((n, T, n \times T)\) the principal normal curvature vector \( \kappa \) is in the plane spanned by \( n \times T \) and \( n \), and can thus be decomposed into these two orthogonal vectors:

\[
\kappa = \kappa_g(n \times T) + \kappa_n n
\]  
(2)

where \( n, T \) denote the unit normal and tangent vector, respectively; \( \kappa_g \) gives the component of the principal curvature vector normal to the surface at a point and is called the normal curvature; \( \kappa_n \) gives the component of the curvature vector tangential to the surface and is called the geodesic curvature.

![Figure 1: An infinitesimal elementary piece of a fiber curve](image)
For an infinitesimal arclength the magnitudes of the tension at both sides of \( P_0 \) and \( P_1 \) are considered equal:

\[
|F_0| = |F_1| = \ell
\]  

The equilibrium equations of the forces exerting on \( P_0 \) and \( P_1 \) in directions normal and tangent to the surface, can be formulated as:

\[
[(F_0 + F_1) \cdot n] n + F_n = 0
\]  

\[
[(F_0 + F_1) \cdot (n \times T)] (n \times T) + F_f = 0
\]

Substituting Eq. (3) into (4) and (5) followed by multiplying both sides of the resulting equations by the vector \( n \) and \( n \times T \), respectively, results in:

\[
F_n = -\kappa_n \ell ds n
\]

\[
F_f = -\kappa_g \ell ds (n \times T)
\]

To prevent fiber sliding on the supporting surface, the friction force \( F_f \) should always be less than the maximum static friction between the supporting surface and the fiber bundle:

\[
|F_f| \leq \mu_{\text{max}} |F_n|
\]

where \( \mu_{\text{max}} \) is the coefficient of maximum static friction between the fiber and the mandrel surface or between the fiber and the previously overwound layer.

Substitution of Eqs. (6) and (7) into (8) gives the non-slippage criterion:

\[
\lambda = \kappa_g / \kappa_n \leq \mu_{\text{max}}
\]

The slippage coefficient \( \lambda \) is defined as the ratio of the geodesic curvature to the normal curvature, which represents the slippage tendency between the fiber bundle and the supporting surface.

In addition, the fibers may bridge on the concave surface and lose contact with the mandrel surface unless the normal force \( F_n \) is acting in the same direction to the normal vector \( n \):

\[
F_n \cdot n \geq 0
\]

Substituting Eq. (6) into (10) and taking into account that the product \( \ell k ds \) is constantly greater than 0, leads to:

\[
k_n \leq 0
\]

Eq. (11) provides the non-bridging criterion of fiber trajectories on a surface. It should be noted that in general the fiber bridging can be eliminated by modifying the winding angles.

3 INTEGRATED DESIGN AND OPTIMIZATION

The design of fiber trajectories is based on two criteria: optimal laminate structure and suitable winding patterns. The fiber trajectories should be able to fulfill design requirements for either strength or manufacturability, e.g. maximum structural performance, maximum critical strength, uniform stress distribution, non-slippage, non-bridging, uniform and full coverage. Please note that excellent structural performance is always the primary goal in the design of composite pressure vessels. A constrained optimization problem can be represented as follows [24]:

\[
\begin{align*}
\text{Min} \ P F (X) \\
\text{Subject to} \ C_i(X) \leq 0 & \quad i = 1, 2, \ldots, m \\
G_k(X) = 0 & \quad k = 1, 2, \ldots, n \\
X_L \leq X \leq X_U
\end{align*}
\]

where \( PF \) stands for the structural performance factor; \( X \) is the vector of design variables; \( X_L \) and \( X_U \) are the lower and upper limits of the design variables, respectively. The cross-sectional shape, winding angle, stacking sequence, slippage coefficient and layer thickness, etc, can be considered as the design variables. \( PF(X) \) is the objective function, \( C_i(X) \) and \( G_k(X) \) denote the inequality and equality constraint functions, respectively. There are two classes of constraints: explicit and implicit constraints. For example, the non-bridging and non-slippage criteria are typical explicit constraints which are explicitly expressed in term of the design variables; the Tsai-Wu criterion and the full coverage condition are typical implicit constraints which cannot be explicitly expressed in term of the design variables. A number of possible indices, e.g. performance factor, structural weight, burst pressure, strain energy density, critical bulking load, can be considered as the optimization objectives by which the composite structural efficiency can be improved. The flow chart of a typical integrated design procedure for filament-wound composite pressure vessels is shown in Fig. 2.
4 GEODESIC-ISOTENSOIDS

4.1 Domes

The domes can represent the general class of shells of revolution. The geometry and applied loads of a dome meridian is schematized in Fig. 3. $R$ and $r_0$ are the radius of the equator and the radius of the polar opening, respectively; $p$ is a uniformly distributed internal pressure and $A$ is an externally applied axial line load. $S(\theta, z)$ represents the vector of a generic shell of revolution in polar coordinates, given by:

$$S(\theta, z) = \{r(z)\cos \theta, r(z)\sin \theta, z\}$$

(12)

where $\theta$ denotes the angular coordinate in parallel direction; $r$ and $z$ represent the radial and axial distance, respectively.
The differential equation for describing isotensoid dome meridian profiles can be given by [25]:

\[
\rho' = \sqrt{\frac{(a + 1)^2 (\rho^2 - \rho_0^2)}{\rho^2 (a + \rho^2)^2 (1 - \rho_0^2)}} - 1
\]  \hspace{1cm} (13)

in which:

\[\rho = \frac{r}{R}, \quad \rho_0 = \frac{r_0}{R}, \quad \zeta = \frac{z}{R}, \quad a = \frac{A}{\pi p R^2}\]  \hspace{1cm} (14)

Eq. (13) provides isotensoid-based cross-sectional shapes of the domes for various \{a, \rho_0\} values. For a given dimensionless opening radius \(\rho_0\), the resulting cross-sectional profile will strongly depend on the \(a\)-value. Depending on the magnitude of the axial forces as related to the internal pressure, several isotensoid meridian profiles (\(\rho_0 = 0.4\)) can be depicted in Fig. 4.

Figure 3: Loads and geometry of a dome section

Figure 4: Isotensoid-based cross-sectional shapes of the domes for various \(a\)-values (\(\rho_0 = 0.4\))
4.2 Toroids

A toroidal pressure vessel is an axisymmetric shell of revolution with a circular cross-section that does not intersect the axis of revolution. The torus is formed by revolving a circle of radius \( r \) about a circle of radius \( R > r \) lying in an orthogonal plane, as pictured in Fig. 5. \( R \) is the distance between the center of the cross-section and the axis of rotation, and \( r \) is the radius of the tube. The regular parameterization of the torus is given by:

\[
S(\theta, \phi) = \begin{cases} 
(R + r \cos \phi) \cos \theta \\
(R + r \cos \phi) \sin \theta \\
rsin\phi
\end{cases} \quad (0 \leq \phi \leq 2\pi, 0 \leq \theta \leq 2\pi) \tag{15}
\]

where \( \theta \) and \( \phi \) indicate the angular coordinates along the parallel and meridional direction of the torus, respectively.

In Section 4.1, several isotensoid-based dome profiles are obtained using various magnitude of dimensionless axial force \( a \) (recall Fig. 4). When the axial force is sufficiently large for forcing the resulting cross-sectional profile to become closed, the shape of the isotensoid dome becomes an isotensoid toroid. Note that the tensional forces of the fibers that proceed from the polar area towards the equator replace here the theoretically required external axial force \( A \), which is applied on the polar cap [26]. The cross-section shapes of the isotensoid toroids for various polar opening radii, are shown in Fig. 6. It should be noted here that the cross-sectional shapes are quasi-elliptic (not circular). The isotensoid toroid belongs to the class of doubly curved surfaces, and is an interesting alternative for spaces having limited height.
The total mass of the circular and isotensoid toroids is respectively calculated at equal volumes [27]. Fig. 7 illustrates the comparison of the dimensionless mass of isotensoid and circular toroids, as a function of the internal volume. The results show that the isotensoid toroid is consistently lighter than the circular one at any equal volume and internal pressure. The mass values of the isotensoid toroids show about 30% maximal reduction as compared to the circular toroids. It is therefore desirable to employ isotensoid-based cross sections instead of the circular ones for producing filament-wound toroidal pressure vessels.

5 NON-GEODESIC-ISOTENSOIDS

5.1 Domes with unequal polar openings

The geometry of a domed pressure vessel with unequal polar openings is given in Fig. 8. The shapes for this class of structures are similar to oblate spheroids. The vessel here is regarded as the combination of two domes, i.e., part I and part II, where \( r_1 \) and \( r_2 \) are the polar radii of the both domes. A schematic representation of its general profile is shown in Fig. 8. The basic load and geometric parameters are the internal pressure \( p \), the axial load \( A \) (as applied on the dome opening), the equatorial radius \( R \) and the polar opening radii \( r_1, r_2 \).
The governing equations for obtaining non-geodesic-isotensoid cross-sectional shapes of the domes can be formulated by [28]:

$$z'' = -\frac{(1 + z'^2)(S \cdot z'^2 \cdot \tan^2 \alpha \cdot \sqrt{1 + \lambda^2} + 2r^2 \cdot \sec \alpha \cdot \sqrt{1 + z'^2})}{S \cdot r \cdot \sqrt{1 + \lambda^2}}$$  \hspace{1cm} (16)

$$\frac{d\alpha}{dr} = \frac{\tan \alpha}{r} \frac{2\lambda r \sqrt{1 + z'^2}}{S \sqrt{1 + \lambda^2}}$$  \hspace{1cm} (17)

The simultaneous solution of the system of differential equations (16) and (17) will finally provide the isotensoid cross-sections and related non-geodesic trajectories for the domes with unequal polar openings.

The integrated design procedure is here applied to determine an isotensoid pressure vessel with \( r_1 = 1 \) and \( r_2 = 2 \). \( \lambda_1 \) and \( \lambda_2 \) for the both dome parts are taken as the design variables. In the numerical solution procedure a slightly reduced value for the winding angle \( \alpha \) at the polar openings is rather desirable (herein \( \alpha = 0.4999\pi \)), in order to avoid infinity during the solution procedure of the differential equations. After application of the calculation routine, we find two slippage coefficients for the both dome parts, \( \lambda_1 = -0.228 \) and \( \lambda_2 = -0.146 \) [29]. Additionally, substituting the resulting \( \lambda_1 \) and \( \lambda_2 \) into the system of equations (16) and (17) gives the equatorial radius \( R = 10.154 \). Fig. 9 shows the sectional three-dimensional views for the isotensoid pressure vessel designed using the present method with \( r_1 = 1 \) and \( r_2 = 2 \), and the corresponding non-geodesic trajectories that proceed from one pole towards the other. It is revealed that the resulting non-geodesic trajectories of the both dome parts are exactly transited at the equators and satisfy the winding conditions perfectly. The integrated design method is demonstrated to be able to successfully create an isotensoid with unequal polar openings.

Figure 9. The obtained isotensoid with unequal polar openings and its non-geodesic paths

5.2 Toroids

In this section we provide the cross-sectional shapes and related fiber trajectories of non-geodesic-isotensoid toroids. Replacing the geodesic condition by the non-geodesic equation for the isotensoids, leads to the basic equation for the non-geodesic-isotensoids [30]:

$$\rho' = -\frac{\sqrt{C^2 \cdot \cos^2 \alpha - (a + \rho^2)^2}}{a + \rho^2}$$  \hspace{1cm} (18)

where:

$$C = (a + 1) / \cos \alpha_0$$  \hspace{1cm} (19)

By differentiating both sides of Eq. (18) with respect to \( z \), \( \rho'' \) can be obtained by:

$$\rho'' = [\tan^2 \alpha - \frac{2\rho' \rho^2}{(a + \rho^2)(\rho' - \lambda \sin \alpha)}] \cdot \frac{1 + \rho'^2}{\rho}$$  \hspace{1cm} (20)

Substituting Eq. (20) into the non-geodesic equation [30] and plugging the dimensionless parameters defined in Eq. (14), the non-geodesic equation can be simplified as:
Simultaneous solution of the system of differential equations (18) and (21) will finally provide the isotensoid-based cross-sectional profiles and related non-geodesic trajectories of toroids. Eq. (18) has two pairs of real, and one pair of imaginary roots. This expression is only valid for the interval $[\rho_{\text{min}}, 1]$ (selected positive real solutions by setting the argument of the numerator equal to zero). For a given slippage coefficient $\lambda$, the resulting meridian profile will strongly depend on the value of the axial load $a$ and the initial winding angle $\alpha_0$. The $\{a, \alpha_0\}$-parameter set is able to completely determine the cross-sectional shapes of the non-geodesic isotensoids.

For numerically solving the coupling equations (18) and (21), we choose the geometrical parameters at the equator as the initial set of values, consisting of $\rho_0 = 0$, $z_0 = 0$ and $\alpha_0$. In addition, a slightly reduced initial value for $\rho$ is required ($\rho_0 = 0.9999$) in order to avoid singularities during the solution procedure of Eqs. (18) and (21).

Depending on the magnitude of the axial forces $A$ as related to the internal pressure $p$, several non-geodesics-based isotensoid meridian profiles are obtained (Fig. 10) with $\alpha_0 = 5^\circ$ and $\lambda = 0.04$. When the axial force is sufficiently large for forcing the resulting meridian profile to be closed, the shape of the isotensoid becomes here a toroid. Fig. 11 illustrates a 3D sectional view of the obtained isotensoid toroid ($\rho_0 = 0.2$, $\lambda = 0.04$). In Fig. 12 the fiber trajectories for various slippage coefficients are outlined in terms of winding angle developments. The results show that the winding angle varies from a relatively small value around $5^\circ$ at the outer periphery of the toroid to about $50^\circ$ at the inner periphery, and its value has an overall decrease with increasing the slippage coefficient.

\[
\frac{d\alpha}{d\zeta} = -\frac{\rho'}{\rho} \tan \alpha + \frac{2\lambda \rho \rho' \cos \alpha}{(a + \rho^2)(\rho^2 - \lambda \sin \alpha)} \tag{21}
\]
CONCLUSIONS

An integrated design and production method of filament-wound composite structures was outlined in this paper. Domed and toroidal pressure vessels were taken as design examples to demonstrate the reliability and effectiveness of the present method. The netting theory was employed to predict the mechanical behaviors of composite pressure vessels, corresponding to various design scenarios. Non-geodesic trajectories were applied to replace the conventional geodesics, in order to improve the structural performance, and to produce unique vessel shapes, e.g. domes with unequal polar openings. Design approaches for determining geodesic and non-geodesic isotensoid toroids were also presented. The results indicate that the meridian curves of the classical isotensoids can become closed if their axial loads on the poles are sufficiently large. It is also shown that the isotensoid toroid has a significantly lower weight than the circular toroid at any equal volume and internal pressure. In addition, the combination of isotensoids with non-geodesics provides a novel and effective solution to improve the weight efficiency of toroidal pressure vessels as well as to increase the gravimetric and volumetric densities of compressed gaseous storage. The ability for simultaneously improving the structural performance and satisfying the manufacturability of composite pressure vessels becomes here a fact.
ACKNOWLEDGEMENTS

This paper is supported by the National Natural Science Foundation of China (Grant No. 11302168), the Natural Science Basic Research Plan in Shaanxi Province of China (Grant No. 2013JQ6018), the Natural Science Foundation of Hubei Province (Grant No. 2014CFB140), and the Fundamental Research Funds for the Central Universities (Grant No. 143101001).

REFERENCES


