EFFICIENT NONLINEAR MULTISCALE SPECTRAL GFEM APPLIED TO COMPOSITE AEROSPACE STRUCTURES

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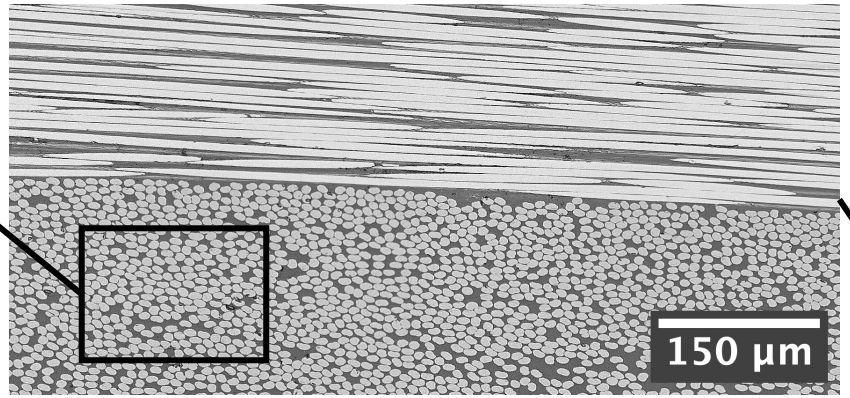
4. *Institute of Data Science and AI, University of Exeter, Exeter, UK,*
5. *digiLab, Exeter, UK.*

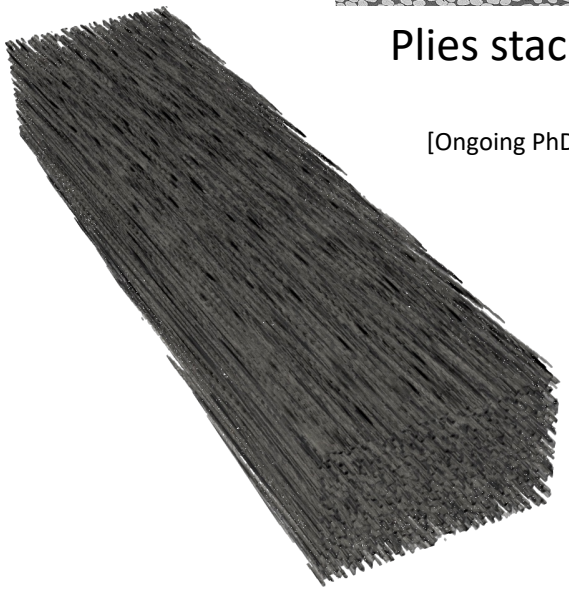
Composite materials for aerospace applications

Inner ply scale:
Microscopic



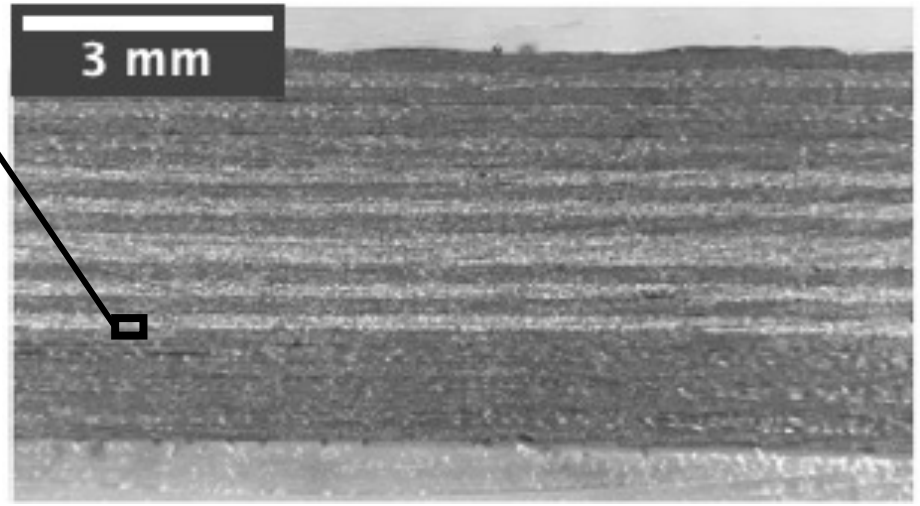
Plies stacked in different orientations

[Ongoing PhD – Jiraphant Srisuriyachot – University of Bath]



Carbon fibres

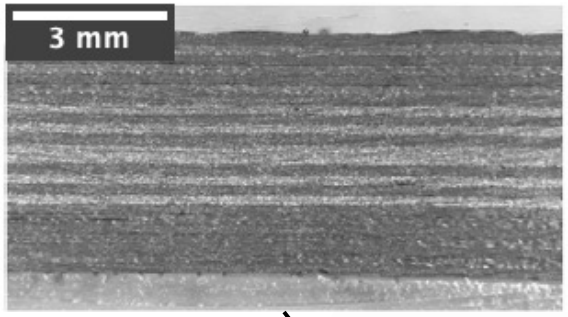
Inner structure scale: **Mesoscopic**



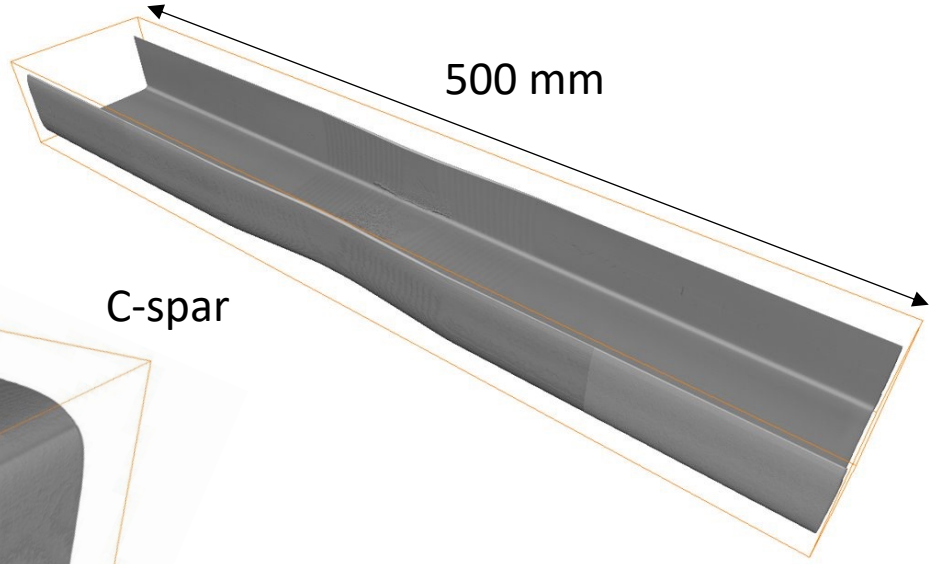
Laminated material

[Johnson, K. J., Butler, R., Loukaides, E. G., Scarth, C., & Rhead, A. T. (2019). Stacking sequence selection for defect-free forming of uni-directional ply laminates. Composites Science and Technology, 171, 34-43.]

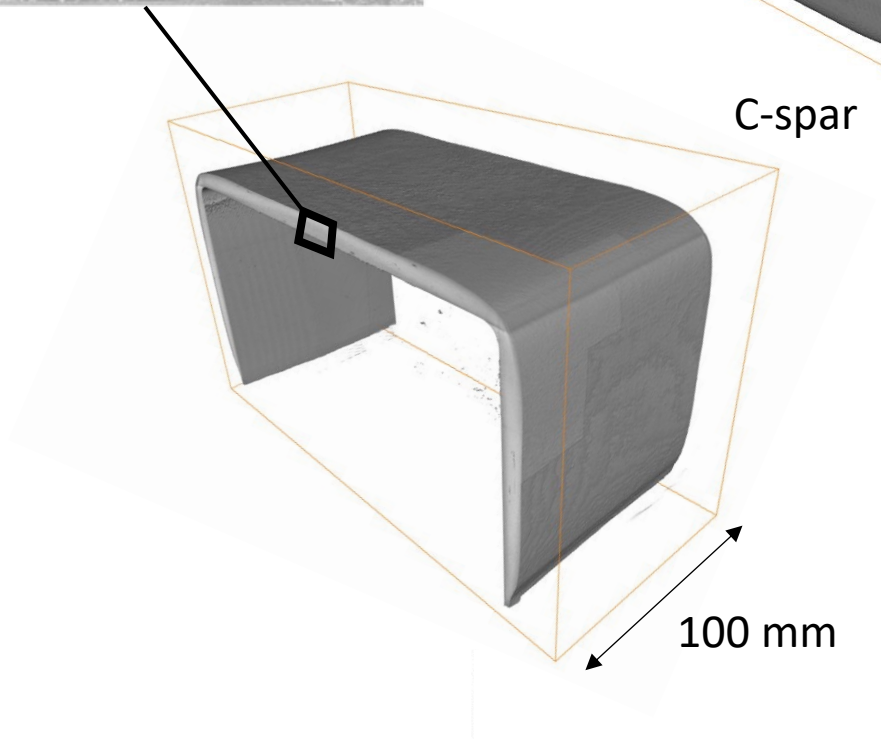
Composite materials for aerospace applications



Structural detail scale



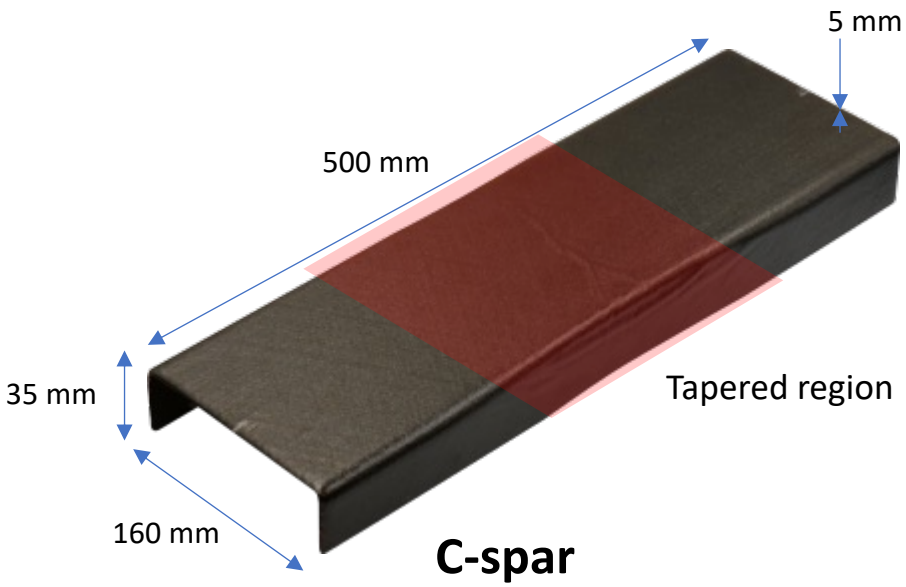
C-spar



Macroscopic scale

Composite fuselage
[GKN Fokker]
https://www.gkn.com/aerospace

Uncertainties:

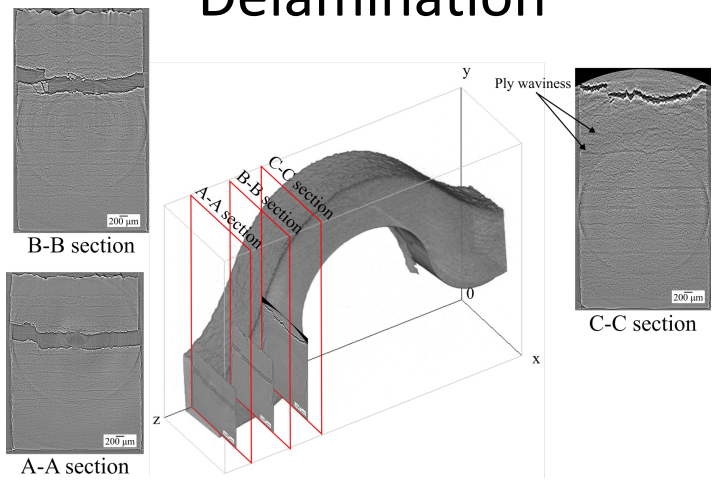


C-spar

Wrinkles

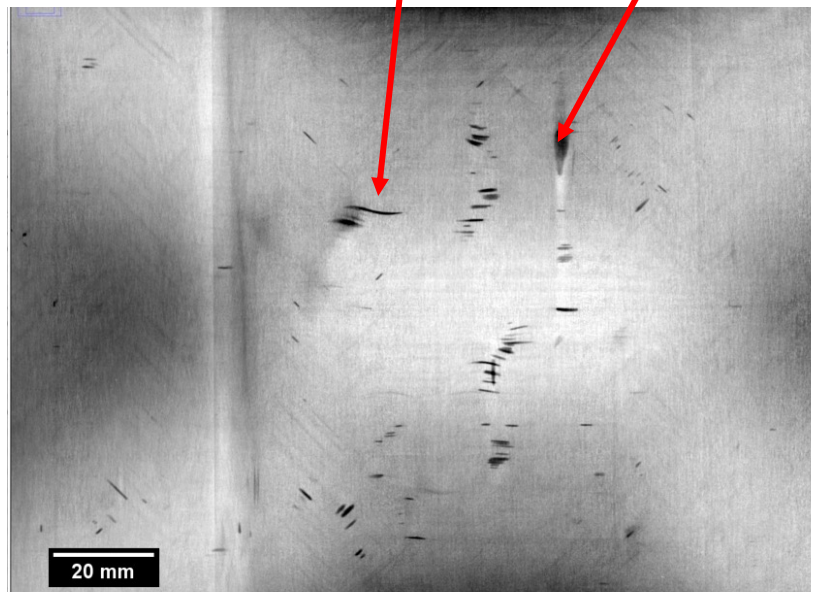
Tapered region
Micro-tomography (µCT) thickness measurement

Delamination



In-plane waviness

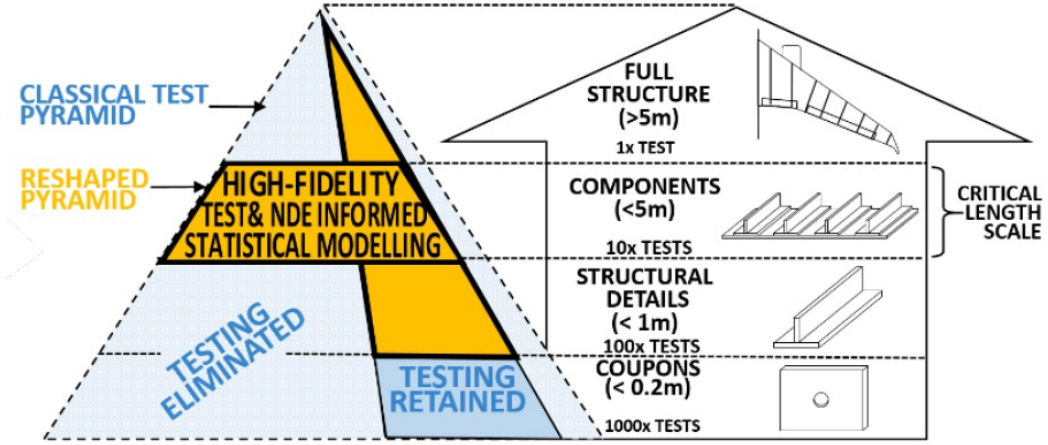
Voids



µCT slice

CERTIFICATION FOR DESIGN: RESHAPING THE TESTING PYRAMID

Design a certification process adapted to composites for aerospace applications



- Multi-scale problem
  - Resolve stress distribution at the defect level
  - Structural effect
  - Nonlinear effect
- Uncertainty quantification (UQ) problem
  - Sub-component and upper levels
  - Virtual testing

Objectives:

- Method designed for UQ – Efficient defect assessment

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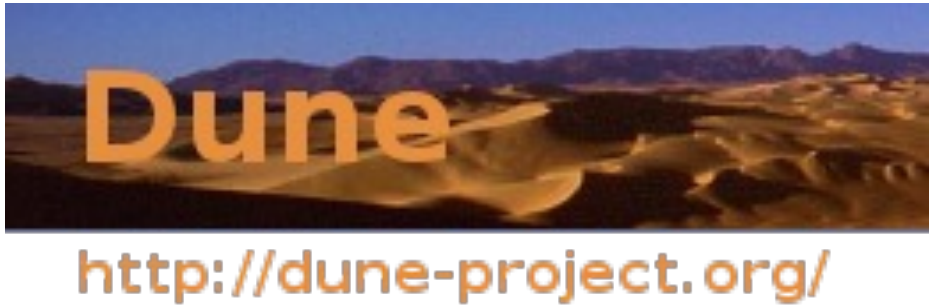
I. MS-GFEM for HPC scale problem & Offline – Online method

II. Non-linear MS-GFEM

III. UM-bridge



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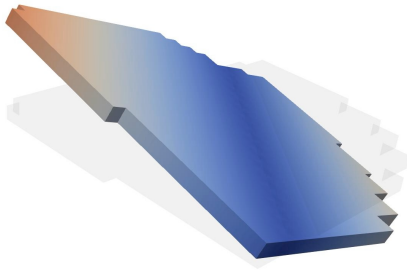
Dune

http://dune-project.org/

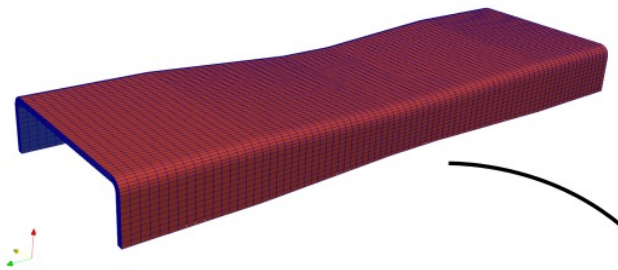
Eigen vector: 0



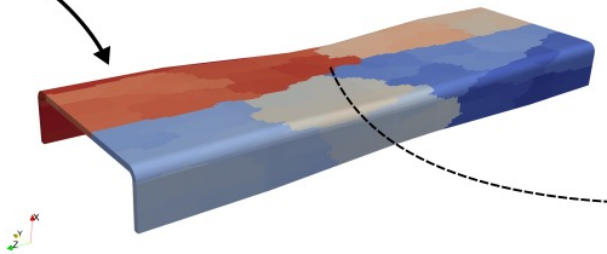
MS-GFEM:

Multi-scale Spectral Generalized Finite Element Method

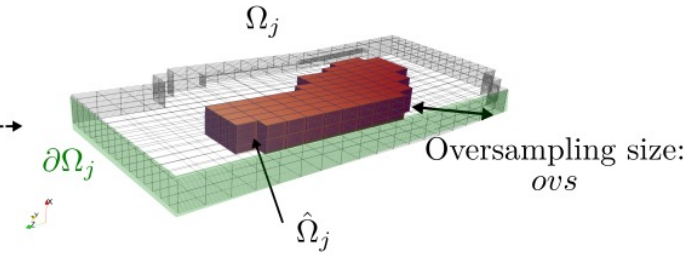
Full meso-scale model



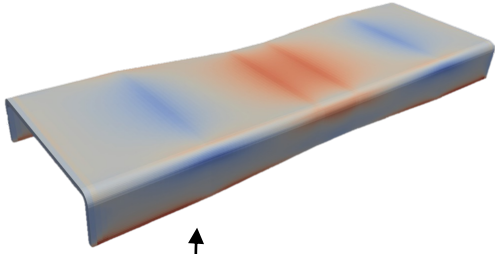
Domain decomposition



Generalized Eigenvalue problem in the Overlaps (GenEO) [1]



Coarse approximation space



A-harmonic optimal subspace [2]:
Ensure that the eigenvectors are solutions of the homogeneous PDE on interior Degrees of Freedom (DoF)

[1] Spillane, N., Dolean, V., Hauret, P., Nataf, F., Pechstein, C., & Scheichl, R. (2014). Abstract robust coarse spaces for systems of PDEs via generalized eigenproblems in the overlaps. *Numerische Mathematik*, *126*(4), 741-770.

[2] Ma, C., & Scheichl, R. (2021). Error estimates for fully discrete generalized FEMs with locally optimal spectral approximations. arXiv preprint arXiv:2107.09988.

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Dune

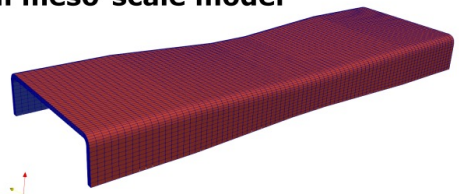
http://dune-project.org/

Offline / Online method:

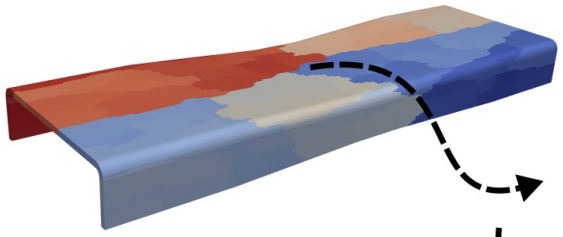
Offline

GMs-FEM
Massively parallel

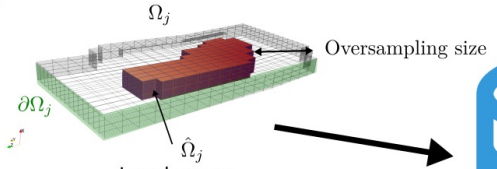
Full meso-scale model

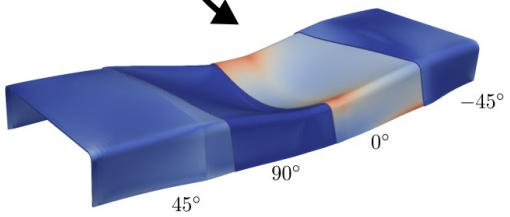


A-harmonic generalized Eigenvalue problem in the Overlaps (GenEO)



Domain decomposition

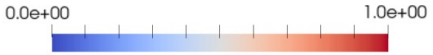




Coarse approximation

Database

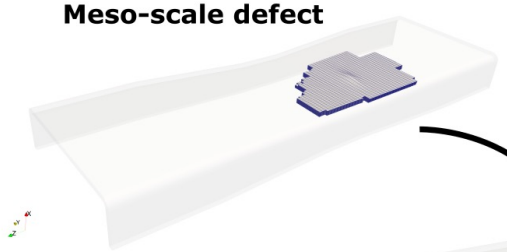
Compressive failure criterion

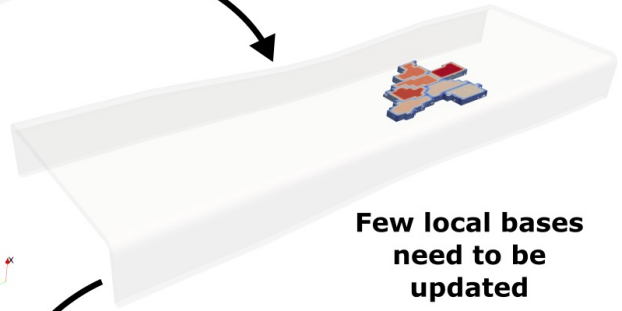


Online

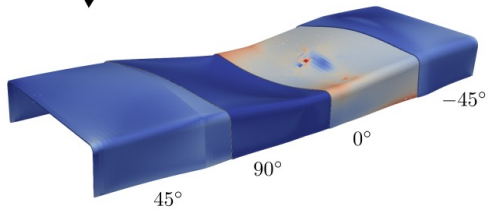
Local update of the coarse space
Computational burden drastically reduced

Meso-scale defect





Few local bases need to be updated



Updated coarse approximation

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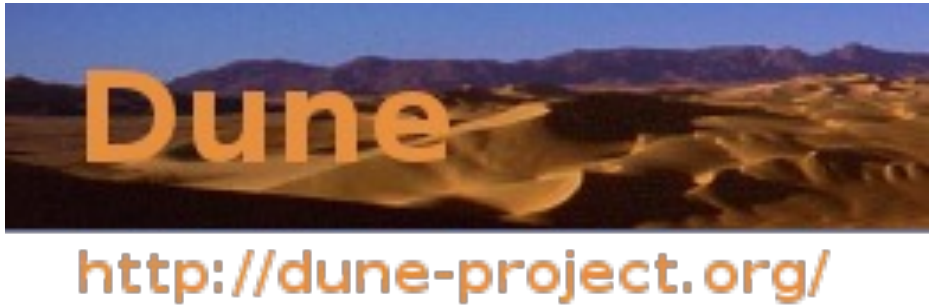
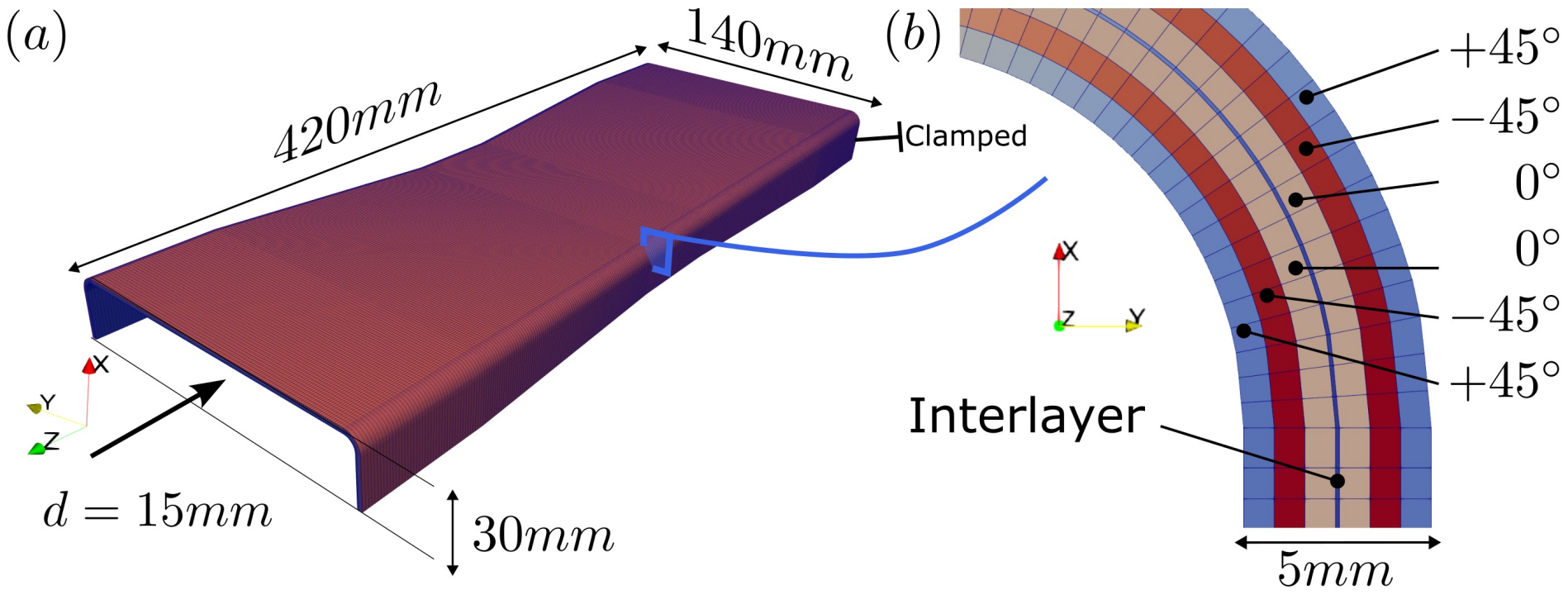
I. MS-GFEM for HPC scale problem & Offline – Online method

II. Non-linear MS-GFEM

III. UM-bridge



Composite problem



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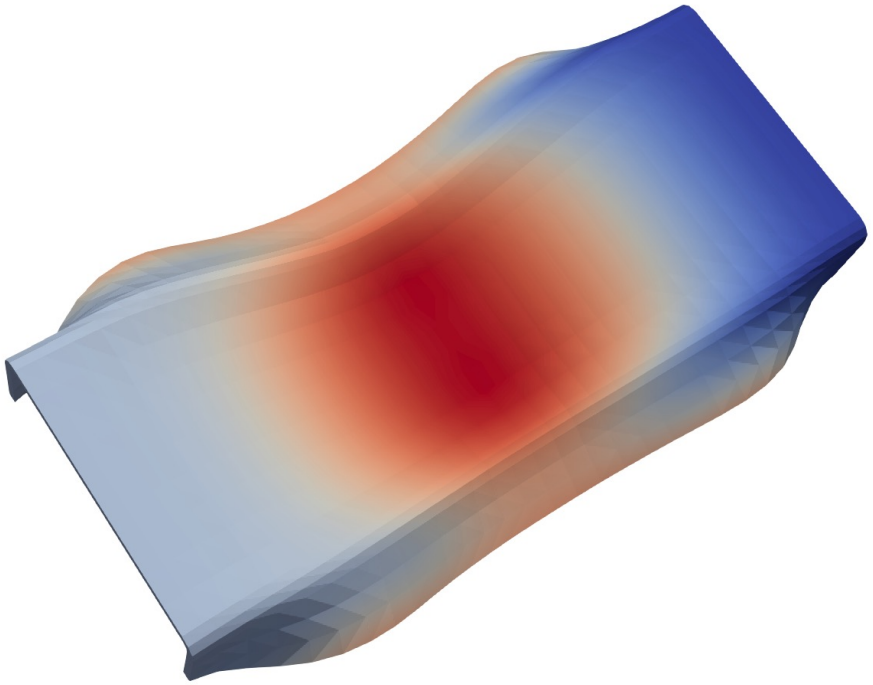
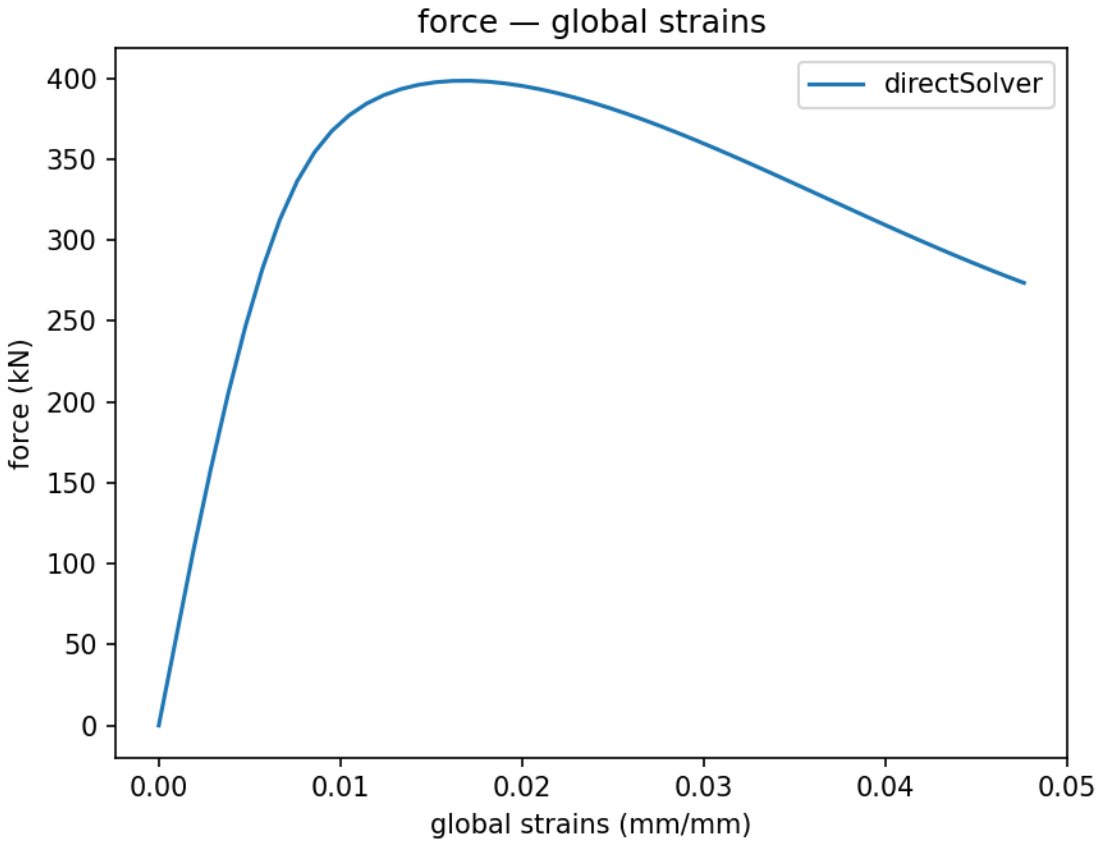
Large thin structure – Buckling



Implement of geometric nonlinearities in DUNE



Cho, H., Shin, S., & Yoh, J. J. (2017). Geometrically nonlinear quadratic solid/solid-shell element based on consistent corotational approach for structural analysis under prescribed motion. *International Journal for Numerical Methods in Engineering*, *112*(5), 434-458.

Nguyen, V. A., Zehn, M., & Marinković, D. (2016). An efficient co-rotational FEM formulation using a projector matrix. *Facta Universitatis, Series: Mechanical Engineering*, *14*(2), 227-240.

Newton Raphson MS-GFEM scheme

Load step loop: $F_{\Omega_i^*\,ext}^{\;t+\delta t} = F_{\Omega_i^*\,ext}^{\;t} + \Delta F_{\Omega_i^*\,ext}$

update $K_{\Omega_i^*}^{T\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

Construct coarse space $V_H := \text{span}\{R_j^{\top}\Xi_j(\varphi_h^{j,k}|_{\Omega_j}) : k = 1, \ldots, m_j, \quad j = 1, \ldots, N\}$

$K_H^{T\,t+\delta t}(u)$

While $||F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}|| < \epsilon \qquad ||\Delta U_H^{t+\delta t}|| < \epsilon$

Solve: Local particular solution: $K_{\Omega_i^*}^{T\;t+\delta t}\Delta u_{\Omega_i^*}^{t+\delta t} = F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}$

Coarse space solution: $K_H^{T\,t+\delta t}\Delta U_H^{t+\delta t} = R_H$

$U_H^{t+\delta t} = U_H^t + \Delta U_H^{t+\delta t}$

Project & update: $u_{\Omega_i^*}^{t+\delta t} \longrightarrow K_{\Omega_i^*}^{T\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

update coarse system: V_H , $K_H^{T\,t+\delta t}(u)$

Improvements: Newton Raphson MS-GFEM scheme

Load step loop: $F_{\Omega_i^*\,ext}^{\;t+\delta t} = F_{\Omega_i^*\,ext}^{\;t} + \Delta F_{\Omega_i^*\,ext}$

update $K_{\Omega_i^*}^{T\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

Construct coarse space $V_H := \text{span}\{R_j^{\top}\Xi_j(\varphi_h^{j,k}|_{\Omega_j}) : k = 1, \ldots, m_j, \quad j = 1, \ldots, N\}$

$K_H^{T\,t+\delta t}(u)$

While $||F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}|| < \epsilon \qquad ||\Delta U_H^{t+\delta t}|| < \epsilon$

Solve: Local particular solution: $K_{\Omega_i^*}^{T\;t+\delta t}\Delta u_{\Omega_i^*}^{t+\delta t} = F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}$

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Project & update: $u_{\Omega_i^*}^{t+\delta t} \longrightarrow K_{\Omega_i^*}^{T\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

update coarse system: V_H , $K_H^{T\,t+\delta t}(u)$

Improvements: Newton Raphson MS-GFEM scheme

Load step loop: $F_{\Omega_i^*\,ext}^{\;t+\delta t} = F_{\Omega_i^*\,ext}^{\;t} + \Delta F_{\Omega_i^*\,ext}$

update $K_{\Omega_i^*}^{T\;\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

Construct coarse space $V_H :=$ span$\{R_j^{\top}\Xi_j(\varphi_h^{j,k}|_{\Omega_j}) : k = 1, \ldots, m_j, \quad j = 1, \ldots, N\}$

$K_H^{T\,t+\delta t}(u)$

While $||F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}|| < \epsilon \qquad ||\Delta U_H^{t+\delta t}|| < \epsilon$

Solve: Local particular solution: $K_{\Omega_i^*}^{T\;\;t+\delta t}\Delta u_{\Omega_i^*}^{t+\delta t} = F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}$

Coarse space solution: $K_H^{T\,t+\delta t}\Delta U_H^{t+\delta t} = R_H$

$U_H^{t+\delta t} = U_H^t + \Delta U_H^{t+\delta t}$

Project & update: $u_{\Omega_i^*}^{t+\delta t} \rightarrow K_{\Omega_i^*}^{T\;\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

update coarse system: V_H , $K_H^{T\,t+\delta t}(u)$

- Reducing load increment
- Improving coarse space
- More local iterations
- Allow re-computation of coarse space if needed

Improvements: Newton Raphson MS-GFEM scheme

Load step loop: $F_{\Omega_i^*\,ext}^{\;t+\delta t} = F_{\Omega_i^*\,ext}^{\;t} + \Delta F_{\Omega_i^*\,ext}$

update $K_{\Omega_i^*}^{T\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

Construct coarse space $V_H := \text{span}\{R_j^{\top}\Xi_j(\varphi_h^{j,k}|_{\Omega_j}) : k = 1, \ldots, m_j, \quad j = 1, \ldots, N\}$

$K_H^{T\,t+\delta t}(u)$

While $||F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}|| < \epsilon \qquad ||\Delta U_H^{t+\delta t}|| < \epsilon$

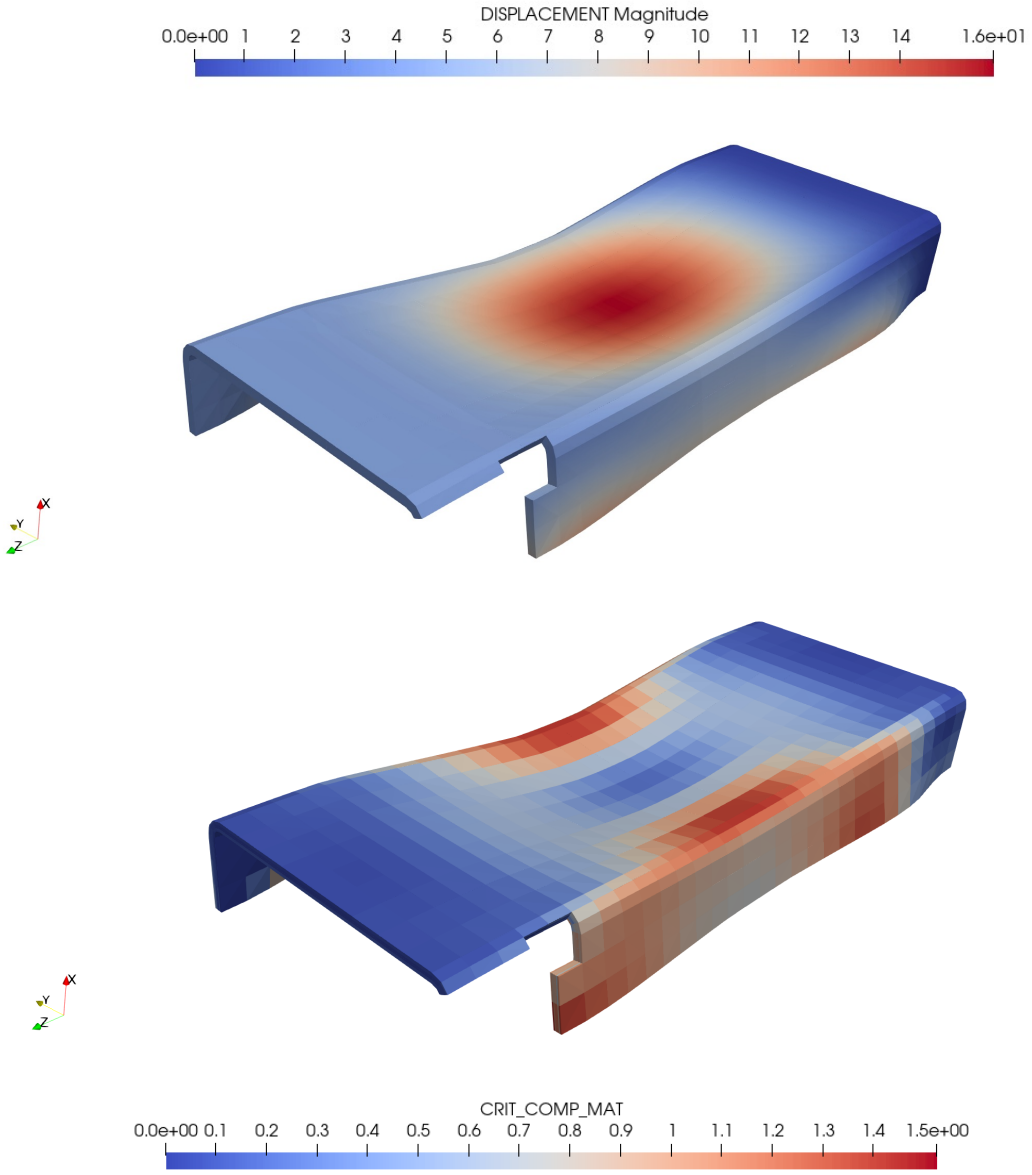
Solve: Local particular solution: $K_{\Omega_i^*}^{T\;t+\delta t}\Delta u_{\Omega_i^*}^{t+\delta t} = F_{\Omega_i^*\,ext}^{\;t+\delta t} - F_{\Omega_i^*\,int}^{\;t+\delta t}$

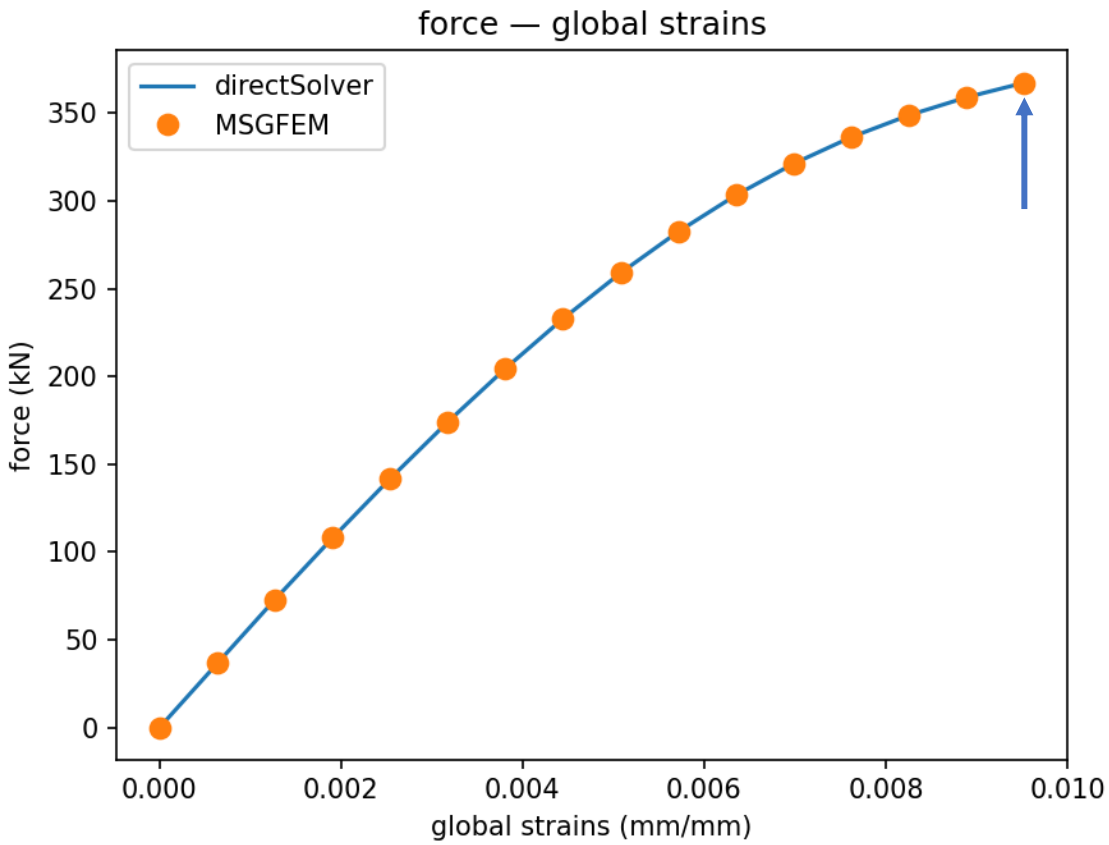
Coarse space solution: $K_H^{T\,t+\delta t}\Delta U_H^{t+\delta t} = R_H$

$U_H^{t+\delta t} = U_H^t + \Delta U_H^{t+\delta t}$

Project & update: $u_{\Omega_i^*}^{t+\delta t} \longrightarrow K_{\Omega_i^*}^{T\;t+\delta t}(u)$, $F_{\Omega_i^*\,int}^{\;t+\delta t}$

Ensure convergence if: $||\Delta U_H^{t+\delta t}||_{i+1} > ||\Delta U_H^{t+\delta t}||_i$ do update coarse system: V_H , $K_H^{T\,t+\delta t}(u)$





Scalability!

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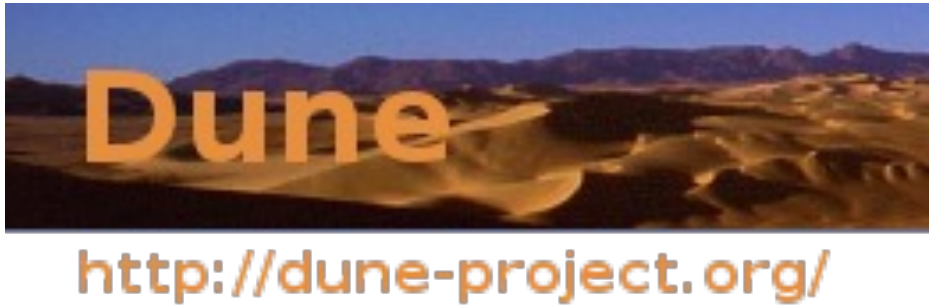

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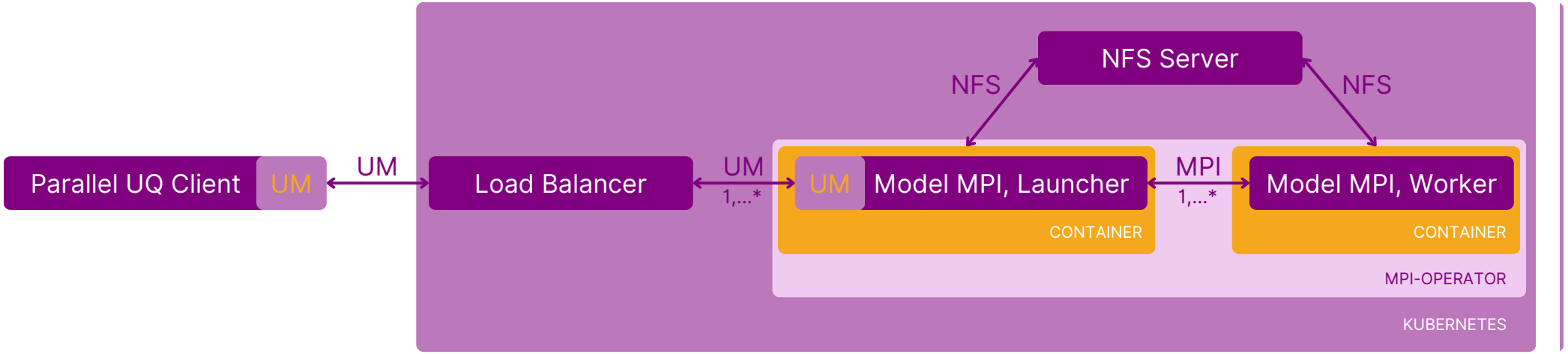
UM-bridge:



https://um-bridge-benchmarks.readthedocs.io/en/docs/index.html

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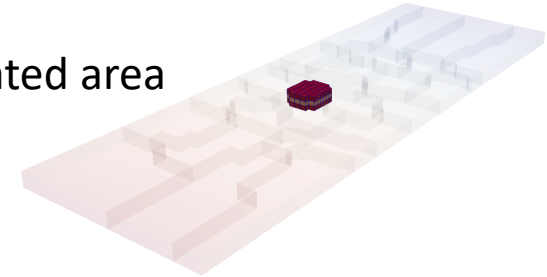
UM-Bridge

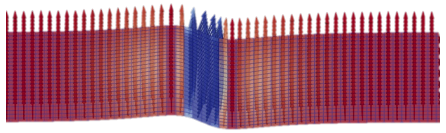
docker

UM-bridge:

Delaminated area



Wrinkles



Inputs:
Vector describing defect (size, angle, type, etc..)

UQ | UM ⇄ HTTP ⇄ UM

Model

Database

Online:
- Unzip database
- Select subdomains
- Update EV basis
- Update local particular solution
- Solve the coarse space

Outputs:
- Failure criterion
- Global response

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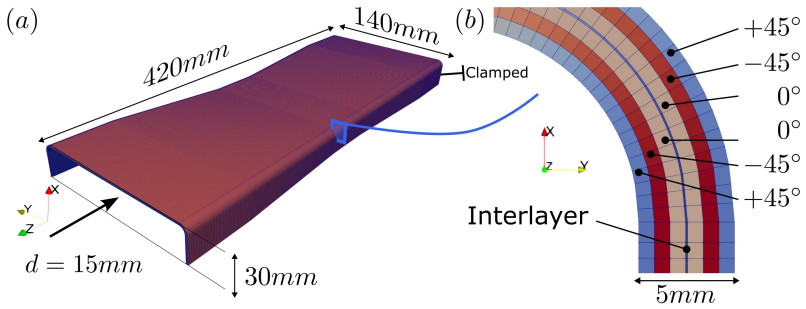
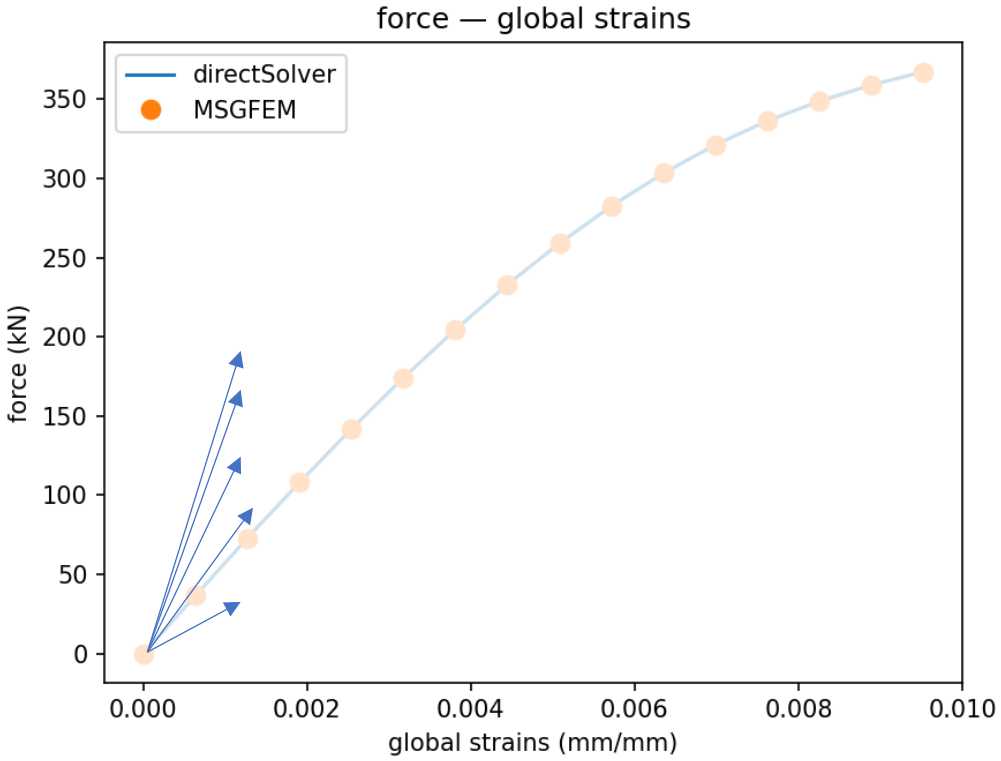
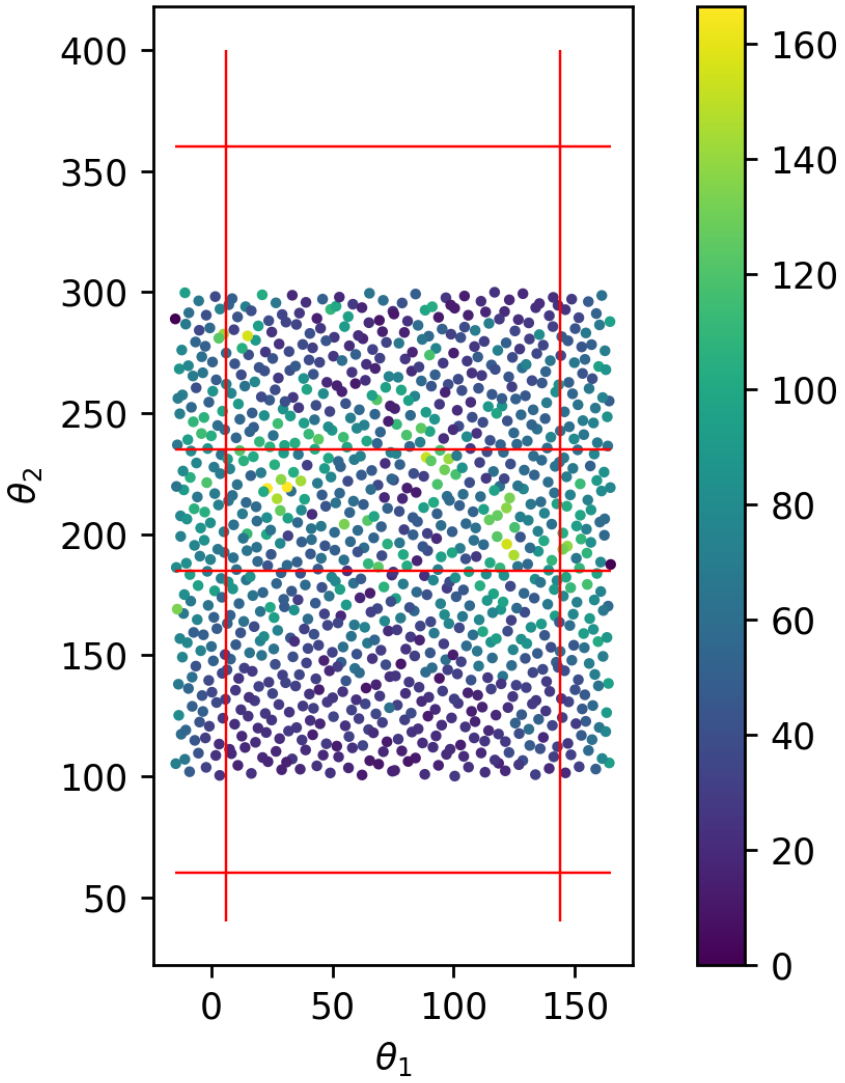
Quasi Monte Carlo:

Delamination defect





Strain energy distribution



< 1h for 1024 samples

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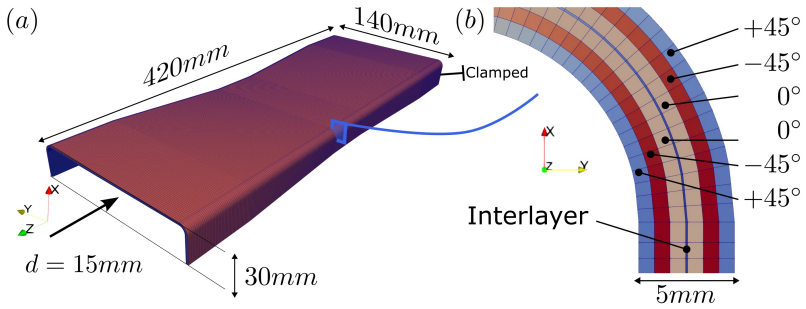
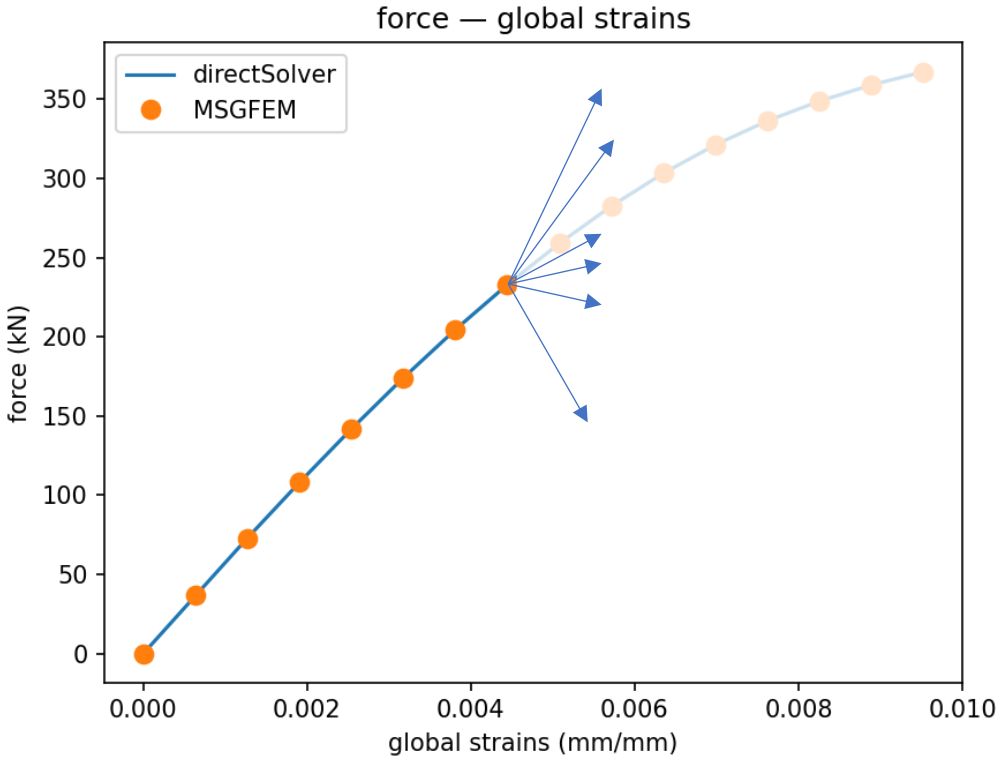
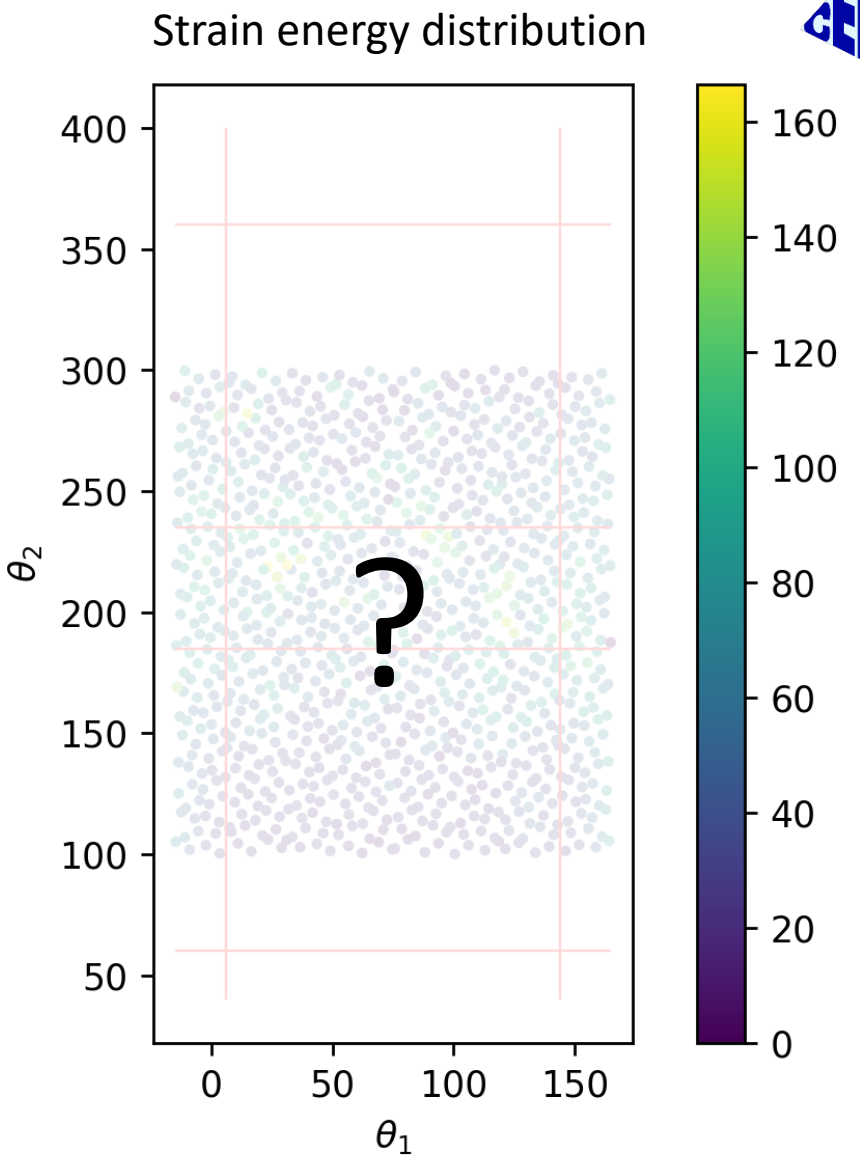
Quasi Monte Carlo:

Start at a load increment

Delamination defect







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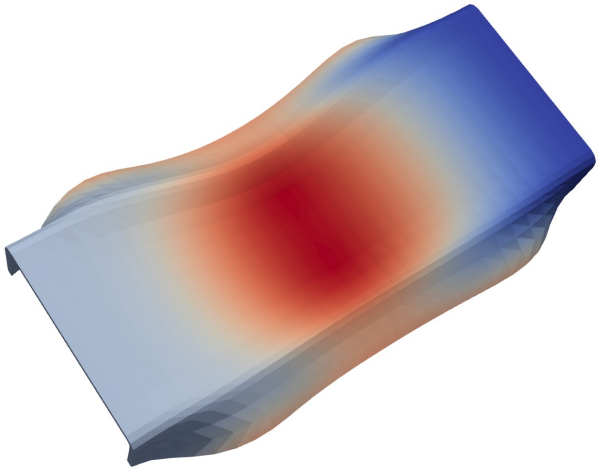
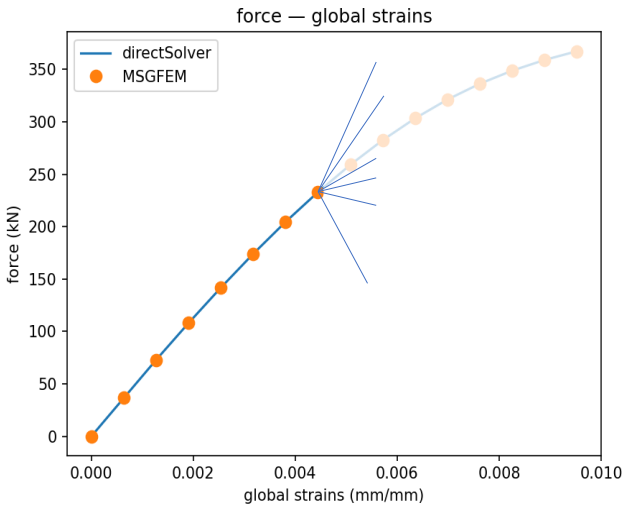
Conclusion:

- Parallelised and optimised nonlinear method with a great scalability



Perspectives:

- Associate Nonlinear MS-GFEM with Offline/Online method
- Singular Value Decomposition



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Thanks for your attention

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